Hořava Gravity in Canonical Formalism

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AstroParticle Physics Seminar
SISSA 2020

Outline

- **Introduction**:
  FDiff symmetry and Hořava theory.
- **Part I**:
  FDiff in the canonical formalism
  A modified model
- **Part II**:
  Quantization with second-class constraints

Hořava theory

- Hořava theory is aimed to be a field theory of quantum gravity

  Unitary + Renormalizable

- Relativistic models of higher curvature

$$\mathcal{L} \sim R^2, R_{\mu\nu}R^{\mu\nu}$$

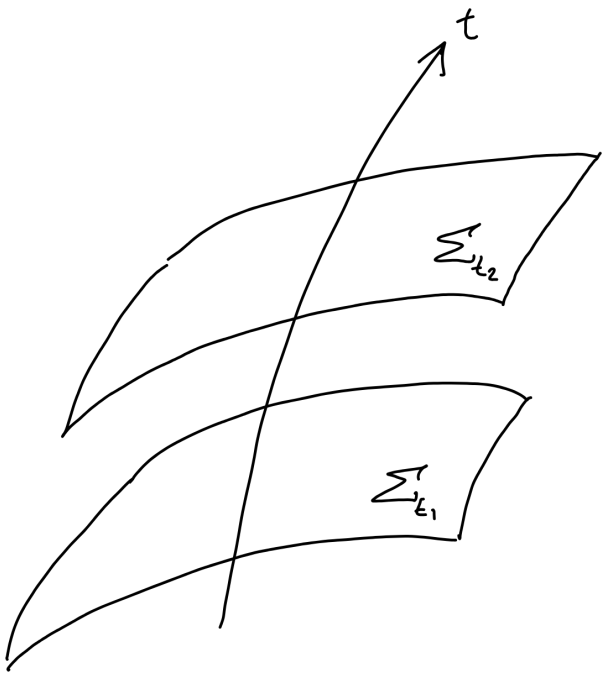
  are renormalizable, but not unitary. The number of propagating degrees of freedom is raised ➜ some of them become ghosts [Stelle 1976].

- **Hořava proposal's**: Suppose that we break the local Lorentz symmetry. The theory can have a different scaling between the time and the space at high energies.
- The arising of ghosts in the spectrum can be forbidden in this way ➜ unitarity can be safe.

[Hořava 2009]

Hořava theory

- **Gravitational space**: there is a line of time with absolute physical meaning, and a foliation of spacelike leaves along it.



- **Symmetry**: The allowed diffeomorphisms are those that preserve the foliation

$$\tilde{t} = \tilde{t}(t)\,, \quad \tilde{x}^i = \tilde{x}^i(t, x^j) \quad \Rightarrow \text{FDiff}$$

Hořava theory

- At high energies, the theory acquires an anistoropic scaling symmetry

$$t \to b^z t\,, \quad x^i \to b x^i$$

- Minimal z necessary for power-counting renormalizability in $3+1$ dimensions: $z = 3$

Lagrangian formulation

- A pseudo-Riemannian space-time metric is not mandatory, there is no general covariance.
- ADM variables:

$$\left.\begin{array}{ll} g_{ij}(t,\vec{x}) & \text{Spatial metric} \\ N(t,\vec{x}) & \text{lapse function} \\ N_i(t,\vec{x}) & \text{shift vector} \end{array}\right\} \quad Z = \int \mathcal{D}g_{ij}\mathcal{D}N\mathcal{D}N_i \times e^{iS}$$

Hořava theory

- Transformation under FDiff:

$$\delta t = f(t)\,, \quad \delta x^i = \zeta^i(t,\vec{x})$$

$$\delta N = \zeta^k \partial_k N + f\dot{N} + \dot{f}N\,,$$
$$\delta N_i = \zeta^k \partial_k N_i + N_k \partial_i \zeta^k + \dot{\zeta}^j g_{ij} + f\dot{N}_i + \dot{f}N_i\,,$$
$$\delta g_{ij} = \zeta^k \partial_k g_{ij} + 2g_{k(i}\partial_{j)}\zeta^k + f\dot{g}_{ij}\,,$$

- N and N_i are regarded as fields over the leaves that evolve in time.
- The Fdiff allow for two possibilities for the lapse function:

Projectable: $N(t)$ Nonprojectable: $N(t,\vec{x})$

$\downarrow$

Preserved by FDiff

- The nonprojectable version has field equations closer to GR (the large-distance effective theory).

The nonprojectable Hořava theory

- Complete, nonprojectable Hořava theory

$$S = \int dt d^3x \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 - \mathcal{V} \right)$$

$K_{ij} = \frac{1}{2N}(\dot{g}_{ij} - 2\nabla_{(i} N_{j)})$ ➜ Extrinsic curvature of the spatial leaves.

$K = g^{ij} K_{ij}$

$\mathcal{V} = \mathcal{V}[g_{ij}, a_i]$ ➜ Most general $z = 3$ potential invariant under FDiff.

$a_i \equiv \frac{\partial_i N}{N}$ ➜ This is a vector of FDiff.

Constant λ: Both $K_{ij} K^{ij}$ and K^2 are independently invariant under FDiff.

[Hořava 2009]
[Blas, Pujolàs , Sibiryakov 2009]

Embedding in Newton-Cartan Geometry

- The gauging of the Galilean algebra leads to the several versions of the Hořava theory.
  - Torsionless connection: Projectable Horava theory.
  - Twistless torsional conection: Nonprojectable theory
  - Central Bargmann extension: $U(1)$ Extension of the Horava theory
- Gauge connection for the Galilean algebra:

$$\mathcal{A}_\mu = \tau_\mu H + e^i_\mu P_i + \Omega_\mu{}^i G_i + \frac{1}{2}\Omega_\mu{}^{ij} J_{ij}$$

Inverse vielbein: $\quad v^\mu \tau_\mu = -1$

- Twistless torsional connection ➜ nonprojectable Horava theory
  Connection with torsion: $\Gamma^\rho_{[\mu\nu]} = -v^\rho \partial_{[\mu}\tau_{\nu]}$
  Twistless condition: $\tau_{[\mu}\partial_\nu \tau_{\rho]} = 0$ ➜ τ_μ is hypersuface orthogonal.

[Hartong, Obers 2015]

Embedding in Newton-Cartan Geometry

- Relation with the foliation and ADM variables:

$$\nabla_\mu v^\nu = -h^{\nu\alpha} K_{\mu\alpha} \quad \rightarrow \text{extrinsic curvature}$$

- The gauging of the Galilean algebra leads to an universal kinetic term

$$e^{-1}\mathcal{L}_K = c_3 \nabla_\mu v^\mu \nabla_\nu v^\nu + c_4 \nabla_\nu v^\mu \nabla_\mu v^\nu = c_4 \left(K_{ij}K^{ij} - \frac{c_3}{c_4} K^2 \right)$$

c_3, c_4 : coupling constants

$$\rightarrow \lambda = \frac{c_3}{c_4}$$

FDiff in canonical formalism

- The FDiff-invariant phase space is spanned by the conjugate pairs (g_{ij}, π^{ij}) and (N, P_N).
- The Lagrangian does not depend on $\dot{N}$, but it depends nonlinearly on N, hence N cannot be a Lagrange multiplier.
- The canonical momentum P_N can be integrated ➜ $P_N = 0$.
- N_i enters as a Lagrange multiplier.
- The Hamiltonian has a "zero" part that is a sum of constraints and a "nonzero" part

$$H = \int d^3x \left(\mathcal{H}_0 + B_A \phi^A\right) + \text{boundary terms}$$

$\mathcal{H}_0$: Nonzero part

ϕ^A : Constraints, B_A: Lagrange multipliers

- One can redefine the Hamiltonian such that it is a total sum of constraints and boundary terms. The bulk Hamiltonian is zero on the constrained phase space ➜ the theory is invariant under $\tilde{t}(t)$.

FDiff in canonical formalism

- We may start from this point. One may write the general form of the canonical action regardless of any original noncanonical Lagrangian.
- FDiff on the variables of the canonical formalism

$$
\begin{aligned}
\delta g_{ij} &= \zeta^k \partial_k g_{ij} + \partial_i \zeta^k g_{kj} + \partial_j \zeta^k g_{ik} + f \dot{g}_{ij} \\
\delta \pi^{ij} &= \zeta^k \partial_k \pi^{ij} - \partial_k \zeta^i \pi^{kj} - \partial_k \zeta^j \pi^{ik} + \partial_k \zeta^k \pi^{ij} + f \dot{\pi}^{ij} \\
\delta N &= \zeta^k \partial_k N + f \dot{N} + \dot{f} N \\
\delta N_i &= \zeta^k \partial_k N_i + \partial_i \zeta^k N_k + \dot{\zeta}^k g_{ik} + f \dot{N}_i + \dot{f} N_i \\
\delta B_A &= \zeta^k \partial_k B_A + f \dot{B}_A + \dot{f} B_A
\end{aligned}
$$

- These FDiff transformations can be implemented in the action if

$$
\begin{aligned}
\delta \mathcal{H}_0 &= \zeta^k \partial_k \mathcal{H}_0 + \partial_k \zeta^k \mathcal{H}_0 + f \dot{\mathcal{H}}_0 + \dot{f} \mathcal{H}_0 \quad \text{density-density} \\
\delta \phi^A &= \zeta^k \partial_k \phi_A + \partial_k \zeta^k \phi_A + f \dot{\phi}_A \quad \text{density-scalar}
\end{aligned}
$$

➜ The canonical action is FDiff-invariant.

FDiff in canonical formalism

- The part of FDiff that is strictly a gauge symmetry are the spatial diffeomorphisms ➜ The momentum constraints is the generator

$$\mathcal{H}^j = -2\nabla_i \pi^{ij} = 0$$

- $\mathcal{H}^i$ is the only first-class constraint, as far as no other gauge symmetries are introduced.
- The canonical action has the general form

$$S = \int dt d^3x \left[\pi^{ij}\dot{g}_{ij} - \left(\mathcal{H}_0 + N_i\mathcal{H}^i + B_A\phi^A\right)\right]$$

- The possibilities for defining canonical theories is broad, since one may define $\mathcal{H}_0$ and the second-class constraints ϕ^A.

Canonical Hořava theory: noncritical case

- The direct Legendre transformation from the Lagrangian of the Hořava theory leads to two cases

$$\mathcal{L} = \sqrt{g}N(G^{ijkl}K_{ij}K_{kl} - \mathcal{V}) \quad \Rightarrow \quad \frac{\pi^{ij}}{\sqrt{g}} = G^{ijkl}K_{kl}$$

$$G^{ijkl} = \frac{1}{2}(g^{ik}g^{jl} + g^{il}g^{jk}) - \lambda g^{ij}g^{kl}\,, \qquad g_{ij}G^{ijkl} = (1 - \lambda d)g^{kl}$$

d: spatial dimension

Case 1. Noncritical: $\lambda \neq 1/d$

$$\mathcal{H}_0 = \frac{N}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{\lambda}{\lambda d - 1}\pi^2\right) + \sqrt{g}N\mathcal{V}\,, \quad \pi = g_{ij}\pi^{ij}$$

- Two second-class constraints: $P_N = 0$ and $\mathcal{H} = 0$.

$$\mathcal{H} = \frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \frac{\lambda}{\lambda d - 1}\pi^2\right) + \frac{\delta}{\delta N}\int d^3x\sqrt{g}N\mathcal{V}$$

$d = 3$ spatial dimensions ➜ three propagating modes: two tensorial transverse modes and one scalar.

Canonical Hořava theory: critical case

Case 2. Critical: $\lambda = 1/d$

$$\mathcal{H}_0 = \frac{N}{\sqrt{g}}\pi^{ij}\pi_{ij} + \sqrt{g}\mathcal{V}$$

$$\mathcal{H} = \frac{1}{\sqrt{g}}\pi^{ij}\pi_{ij} + \frac{\delta}{\delta N}\int d^3x\sqrt{g}N\mathcal{V} = 0$$

- Additional primary constraint

$$g_{ij}G^{ijkl} = 0 \quad \Rightarrow \quad \pi \equiv g_{ij}\pi^{ij} = 0$$

  associated (Dirac procedure) with another extra constraint

$$\mathcal{C} = \frac{3}{2\sqrt{g}}\pi^{ij}\pi_{ij} - \frac{g_{ij}}{N}\frac{\delta}{\delta g_{ij}}\int d^3x\sqrt{g}N\mathcal{V} = 0$$

- Four second-class constraints: P_N, $\mathcal{H}$, π and $\mathcal{C}$.
- $d = 3$ spatial dimensions ➜ two propagating degrees of freedom: transverse tensorial modes.

Critical case: large-distance model

- Potential for large distances

$$\mathcal{V} = -\beta R - \alpha a_i a^i\,, \quad R : \text{Spatial Ricci sc.}, \quad a_i = \partial_i \ln N$$

- Constraints and EOMs

$$P_N = 0\,, \quad \frac{1}{\sqrt{g}}\mathcal{H} = \frac{1}{g}\pi^{ij}\pi_{ij} - \beta R + 2\alpha\frac{\nabla^2 N}{N} - \alpha a_i a^i = 0$$

$$\pi = 0\,, \quad \frac{1}{\sqrt{g}}\mathcal{C} = \frac{3}{2g}\pi^{ij}\pi_{ij} + \frac{\beta}{2}R - 2\frac{\nabla^2 N}{N} + \frac{\alpha}{2}a_i a^i = 0$$

$$\dot{g}_{ij} = \frac{2N}{\sqrt{g}}\pi_{ij} + \mu g_{ij}$$

$$\frac{\dot{\pi}^{ij}}{\sqrt{g}} = -\frac{N}{g}(2\pi^{ik}\pi_k{}^j - \frac{1}{2}g^{ij}\pi^{kl}\pi_{kl}) - \mu\frac{\pi^{ij}}{\sqrt{g}}$$

$$+\left[\beta(\nabla^{ij} - g^{ij}\nabla^2) - \beta(R^{ij} - \frac{1}{2}g^{ij}R) - \alpha(a^i a^j - \frac{1}{2}g^{ij}a_k a^k)\right]N$$

Critical case: large-distance model

- Critical case: the large-distance effective theory propagates gravitational waves of the same kind of GR.
- Generation of the waves: the dominant mode is the same of GR

$$h_{ij}^{TT} = \frac{\kappa_{\mathrm{H}}}{4\pi\beta r} P_{ijkl}^{TT} \frac{d^2 I_{kl}(t - r/\sqrt{\beta})}{dt^2}\,, \qquad I_{kl} = \int d^3x x^k x^l \rho$$

mass quadrupole

[JB, Restuccia 2017] [Ramos, Barausse 2018; Barausse 2019]

- Cosmological scales: the problem of homegenous isotropic configurations. Perfect fluid

$$T_{\mu\nu} = 8\pi G\left(\rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu)\right)\,, \qquad \text{at rest: } u^\mu = (N^{-1}, 0, 0, 0)$$

- The rules for the coupling of the fluid can be taken from GR

$$\frac{\delta}{\delta N} \to \sqrt{g}\rho\,, \qquad \frac{\delta}{\delta g_{ij}} \to \sqrt{g} N g^{ij} P$$

- Homogeneity+isotropy: $N = 1$, $g_{ij} = a^2(t)\delta_{ij}$ ➜ the only posibility is $\rho = 0$ and $a =$ constant.

Modifying the critical theory

- The possibility of defining the theory starting from the canonical formalism allows to modify the critical theory
- We may keep a similar structure of constraints as in the critical theory, such that we define a theory with two degrees of freedom.
- Most general $\mathcal{H}_0$ that

  - Transforms under FDiff as required (double density)
  - Has separate kinetic and potential parts
  - Quadratic in π^{ij}

$$\mathcal{H}_0 = \frac{N}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \omega\pi^2\right) + \sqrt{g}N\mathcal{V}\,, \quad \pi = g_{ij}\pi^{ij}$$

ω : coupling constant

$\mathcal{V}$: general FDiff-invariant potential of order $z = 3$

- Hamiltonian constraint $\Leftarrow \dot{P}_N = 0$

$$\mathcal{H} \equiv \frac{\delta}{\delta N}\int d^3x\mathcal{H}_0 = \frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \omega\pi^2\right) + \frac{\delta}{\delta N}\int d^3y\sqrt{g}N\mathcal{V} = 0$$

Modifying the critical theory

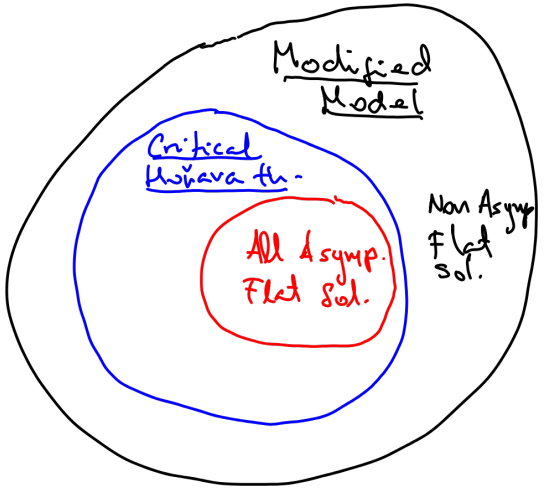
- Additional second-class constraints

$$\begin{aligned}\phi^1 &= \nabla^2\pi = 0\\ \phi^2 &= \frac{1}{N}\left\{\phi^1, \int d^3x\mathcal{H}_0\right\}\\ &= N^{-1}\nabla^2\left(\frac{3N}{2}\frac{\delta}{\delta N} + g_{ij}\frac{\delta}{\delta g_{ij}}\right)\int d^3y\sqrt{g}N\mathcal{V} = 0\end{aligned}$$

- The model has 4 second-class constraints $P_N = 0$, $\mathcal{H} = 0$, $\phi^1 = 0$ and $\phi^2 = 0$. The first-class constraints is $\mathcal{H}^i = 0$.

Relation with the critical Hořava theory:

- The critical Hořava theory is contained ➜ any solution of the constraints and field equations of the critical Hořava theory is a solution of this modified model.
- Asymptotically flat configurations of both theories are exactly the same.



Large-distance effective theory

- Effective potential

$$\mathcal{V}^{(z=1)} = -\beta R - \alpha a_k a^k$$

$$\mathcal{H}_0 = \frac{N}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \omega\pi^2\right) - \sqrt{g}N\left(\beta R + \alpha a_k a^k\right) ,$$

$$\begin{aligned}
\mathcal{H}^i &= -2\nabla_k \pi^{ik} = 0 \\
\mathcal{H} &= \frac{1}{\sqrt{g}}\left(\pi^{ij}\pi_{ij} - \omega\pi^2\right) - \sqrt{g}\left(\beta R + \alpha a_k a^k - 2\alpha\frac{\nabla^2 N}{N}\right) = 0 \\
\phi^1 &= \nabla^2\pi = 0 \\
\phi^2 &= \sqrt{g}N^{-1}\nabla^2(\beta N R + \alpha N a_k a^k) - \gamma_2\sqrt{g}N^{-1}\nabla^4 N = 0
\end{aligned}$$

$$\gamma_2 \equiv \beta + 3\alpha/2$$

Gravitational waves

- Perturbative variables

$$g_{ij} = \delta_{ij} + h_{ij} \quad \pi^{ij} = p_{ij} \quad N = 1 + n$$

$$h_{ij} = h^{TT}_{ij} + \frac{1}{2}\left(\delta_{ij} - \frac{\partial_{ij}}{\Delta}\right) h^T + \partial_{(i} h^L_{j)}$$

h^L_i ➜ Gauge fixed (transverse gauge)
h^T, n ➜ Fixed by second-class constraints $\mathcal{H}$, ϕ^2

$$\left.\begin{array}{l} \mathcal{H} = 0 \\ \phi^2 = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \Delta\left(\beta h^T + 2\alpha n\right) = 0 \\ \Delta^2\left(\beta h^T + \gamma_2 n\right) = 0 \end{array}\right\} \Rightarrow h^T = n = 0$$

p^L_i ➜ Fixed by the first-class constraint $\mathcal{H}^i$
p^T ➜ Fixed by the second-class constraint ϕ^1

$$\phi^1 = 0 \Rightarrow \Delta p^T = 0 \Rightarrow p^T = 0$$

- Wave equation with wave speed $\sqrt{\beta}$

$$\ddot{h}^{TT}_{ij} - \beta \Delta h^{TT}_{ij} = 0 \quad \text{➜ the model is dynamically consistent}$$

Homogeneity and isotropy

- Use the same rule to couple to perfect fluid

$$T_{\mu\nu} = 8\pi G\left(\rho u_\mu u_\nu + P(g_{\mu\nu} + u_\mu u_\nu)\right) , \quad u^\mu = (N^{-1}, 0, 0, 0)$$

$$\frac{\delta}{\delta N} \to -16\pi G\sqrt{g}\rho , \quad \frac{\delta}{\delta g_{ij}} \to +8\pi G\sqrt{g}Ng^{ij}P$$

- The EOMs δN, δg_{ij} and the constraints $\mathcal{H}$, ϕ^2 get contributions from the source (they are defined by means of variations of the Hamiltonian)
- Homogeneity and isotropy

$$N = 1 , g_{ij} = a^2(t)\delta_{ij},$$
$$\pi^{ij}, P, \rho \to \text{functions of time.}$$

➜ Constraints $\mathcal{H}^i, \phi^1, \phi^2$ are automatically solved.

Homogeneity and isotropy

- From the EOM variation $\delta\pi^{ij}$ we get

$$\pi^{ij} = \left(\frac{1}{1-3\omega}\right)\dot{a}\delta_{ij}$$

Combining with the EOM δg_{ij} and constraint $\mathcal{H}$ we get a system analogous to the Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_c}{3}\rho\,, \quad \frac{\ddot{a}}{a} = -\frac{4\pi G_c}{3}(\rho + 3P)$$

$$G_c \equiv 2(3\omega - 1)G$$

➜ Dynamical universes can be modelled in this theory.

Some observational bounds

The observational bounds are imposed on the large-distance effective action $z = 1$.

- Speed of gravitational waves: $c_T = \sqrt{\beta}$. Bounds on the observed waves put c_T extremely close to the speed of light

  $$-3\times10^{-15} \leq c_T - 1 \leq 7\times10^{-16}$$ [LIGO 2017] ➜ Strongly suggests $\beta = 1$

- PPN analysis can be taken directly from Hořava theory since this deals only with asymptotically flat configurations [Blas, Sanctuary 2011] [Foster, Jacobson 2006 for Einstein-aether theory] ➜ Asymptotically flat solutions are the same.
- The PPN parameters coincide with those of general relativity, except for α_1^{PPN}, α_2^{PPN} ➜ these indicate preferred-frame effects!

  $$\alpha_2^{\text{PPN}} = \frac{1}{8}\alpha_1^{\text{PPN}} = \beta - 1 - \frac{\alpha}{2}$$

  Stringtest bound: $\alpha_2^{\text{PPN}} < 10^{-9}$ [Will 2014]. With $\beta = 1$ ➜ Strongly suggests $\alpha = 0$

Quantization with second-class constraints

- The projectable version, $N(t)$, of the Hořava theory has only first-class constraint. The renormalization of this theory has been proven [Barvinsky et al. 2015].
- The nonprojectable Hořava theory has second-class constraints
- In the noncritical case, $\lambda \neq 1/d$, the 2nd class constraints are $P_N = 0$ and $\mathcal{H} = 0$.
- With second-class constraints θ_i the quantization can be achieved with the Senjanovic measure in the path integral

$$\sqrt{\det\{\theta_i, \theta_j\}} \qquad \{\,,\,\} \rightarrow \text{Poisson brackets}$$

Quantization with second-class constraints: 2+1

- Explicit computations in $2+1$ dimensions: Noncritical formulation has 1 scalar degree of freedom (critical $2+1$ has no degrees of freedom).
- Nonprojectable, noncritical theory in $2+1$

$$S = \int dt d^2x \sqrt{g} N \left(G^{ijkl} K_{ij} K_{kl} - \mathcal{V} \right)$$

- Full potential [Sotiriou, Visser, Weinfurtner 2011]

$$\begin{aligned} \mathcal{V} &= -\beta R - \alpha a^2 + \alpha_1 R^2 + \alpha_2 a^4 + \alpha_3 R a^2 + \alpha_4 a^2 \nabla_k a^k \\ &\quad + \alpha_5 R \nabla_k a^k + \alpha_6 \nabla^l a^k \nabla_l a_k + \alpha_7 (\nabla_k a^k)^2 \end{aligned}$$

- Hamiltonian constraint

$$\begin{aligned} \mathcal{H} &= \frac{1}{\sqrt{g}} \left(\pi^{ij} \pi_{ij} + \frac{\lambda}{1-2\lambda} \pi^2 \right) + \sqrt{g} \mathcal{V} + \frac{\sqrt{g}}{N} \left[2\alpha \nabla_k (N a^k) \right. \\ &\quad -4\alpha_2 \nabla_k (N a^2 a^k) - 2\alpha_3 \nabla_k (N R a^k) + \alpha_4 \left(\nabla^2 (N a^2) + 2 \nabla_l (\nabla_k a^k N a^l) \right) \\ &\quad \left. + \alpha_5 \nabla^2 (NR) + 2\alpha_6 \nabla^{kl} (N \nabla_l a_k) + 2\alpha_7 \nabla^2 (N \nabla_l a^l) \right] \end{aligned}$$

Quantization with second-class constraints: 2+1

- The matrix of second-class constraints acquires a triangular form

$$\{\theta_p, \theta_q\} = \begin{pmatrix} \{\mathcal{H}, \mathcal{H}\} & \{\mathcal{H}, P_N\} \\ \{P_N, \mathcal{H}\} & 0 \end{pmatrix} \quad \Rightarrow \quad \sqrt{\det\{\theta_p, \theta_q\}} = \det\{\mathcal{H}, P_N\}$$

- We can use ghosts for the Senjanovic measure

$$\sqrt{\det\{\theta_p, \theta_q\}} = \int \mathcal{D}\bar{\eta}\,\mathcal{D}\eta \exp\left(i \int dt d^2x\, \bar{\eta}\,\{\mathcal{H}, P_N\}\,\eta\right)$$

- Propagators can be obtained from the quadratic action. Perturbations

$$g_{ij} = \delta_{ij} + h_{ij}\,, \quad N = 1 + n$$
$$h_{ij} = \left(\delta_{ij} - \frac{\partial_i \partial_j}{\partial^2}\right) h^T + \partial_{(i} h_{j)}$$

- Gauge degrees of freedom can be integrated out

$$h_i^L = 0 \Leftarrow \text{transverse gauge}$$
$$p_i^L = 0 \Leftarrow \text{momentum constraint}$$

Quantization with second-class constraints: 2+1

- Remaining in the measure

$$\delta(\mathcal{H}) \times \sqrt{\det\{\theta_p, \theta_q\}}$$

- Constraint $\mathcal{H}$ can be raised to the Lagrangian with a Lagrange multiplier a

$$\delta(\mathcal{H}) = \int \mathcal{D}a \exp\left(i \int dt d^2x \, a\mathcal{H}\right)$$

- For the Senjanovic measure we can use ghosts $\bar{\eta}, \eta$.
- Resulting path integral:

$$\begin{aligned}\mathcal{Z}_0 \quad = \quad & \int \mathcal{D}h^T \mathcal{D}n \mathcal{D}a \mathcal{D}\bar{\eta}\, \mathcal{D}\eta \exp\left[i \int dt d^2x \left(\frac{1-2\lambda}{4(1-\lambda)} \dot{h}^T \dot{h}^T - \alpha_1 h^T \partial^4 h^T \right.\right. \\ & -(\alpha_6+\alpha_7) n \partial^4 n + \alpha_5 h^T \partial^4 n + a\left(-\alpha_5 \partial^4 h^T + 2(\alpha_6+\alpha_7)\partial^4 n\right) \\ & \left.\left. +2(\alpha_6+\alpha_7)\bar{\eta}\partial^4\eta\right)\right]\end{aligned}$$

Quantization with second-class constraints: 2+1

- Propagators. $M = (1-\lambda)/(1-2\lambda)$, Fourier space: $\omega, \vec{k}$

$$
\begin{aligned}
< h^T h^T > &= 4M\left(\omega^2 + M\left(4\alpha_1 - \frac{\alpha_5^2}{\alpha_6+\alpha_7}\right)k^4\right)^{-1} \\
< nn > &= \frac{\alpha_5^2 M}{(\alpha_6+\alpha_7)^2}\left(\omega^2 + M\left(4\alpha_1 - \frac{\alpha_5^2}{\alpha_6+\alpha_7}\right)k^4\right)^{-1} \\
< h^T n > &= \frac{2\alpha_5 M}{\alpha_6+\alpha_7}\left(\omega^2 + M\left(4\alpha_1 - \frac{\alpha_5^2}{\alpha_6+\alpha_7}\right)k^4\right)^{-1} \\
< h^T a > &= 0 \\
< na > &= < aa > = < \bar{\eta}\eta > = -\frac{1}{(\alpha_6+\alpha_7)k^4}
\end{aligned}
$$

  ➜ nonregular propagators

- In the projectable theory full regular propagators were obtained by using a nonlocal gauge fixing ➜ essential for renormalization [Barvinsky et al. 2015].

Conclusions

- The Fdiff symmetry can be implemented in the canonical action directly. This opens the possibilities of defining more Hořava-like models.
- We have presented a model with self-consistent classical dynamics. The constraints can be solved. It has two tensorial modes that satisfy hyperbolic equations (wave equations).
- The observational constraints we have presented can be satisfied by restricting two coupling constants of the large-distance effective action.
- Further studies are necessary to scrutinize the consistent quantization of models like this. The model has 4 second-class constraints ➜ this leads to the same degrees of freedom of GR.
- We have presented a scheme of quantization of the (original) nonprojectable Hořava theory. The second-class constraints must be considered. Thanks to a simplification, the Senjanovic measure can be incorporated to the lagrangian by means of ghosts.
- In this scheme nonregular propagators persist, in particular for a Lagrange multiplier and the second-class ghosts. Other gauge-fixing conditions are under study.