MAX-PLANCK-GESELLSCHAFT

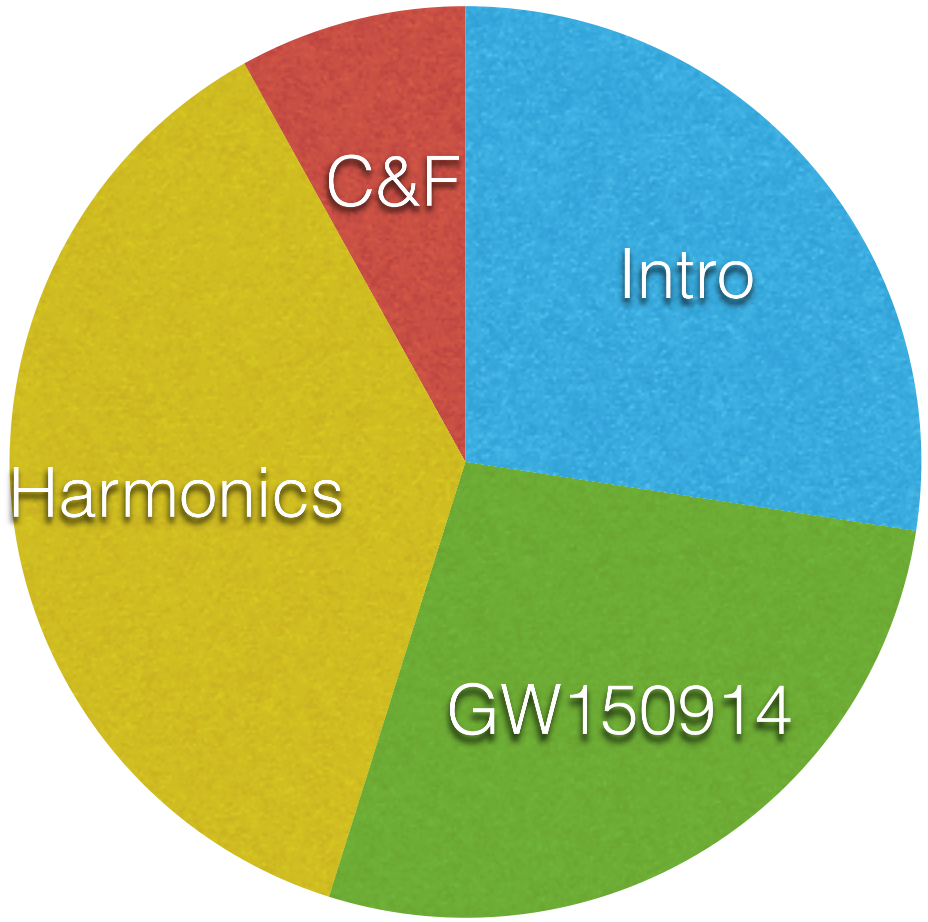
# Ringdown overtones, black hole spectroscopy, and no-hair theorem tests with overtones and angular modes.

Xisco Jiménez Forteza, Swetha Bhagwat, Paolo Pani, Valeria Ferrari

Sissa. July, 2020

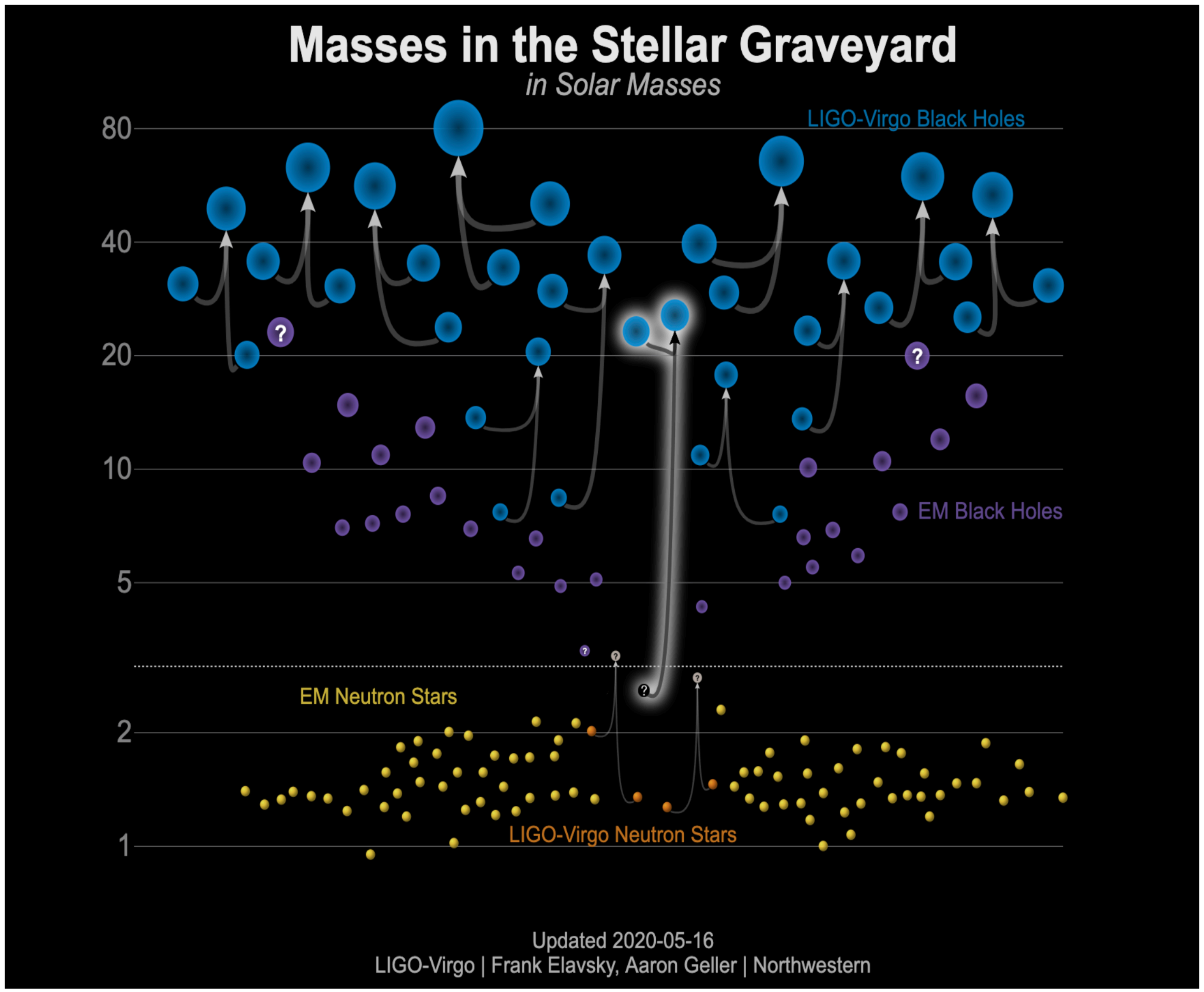
# Outline

- **Introduction.**
    - BBH Observations state-of-the-art.
    - RD modelling premier.
    - RD last results.
- **A RD model for GW150914: overtones.**
    - Fitting to NR data.
    - The systematic error issue.
    - Correlations and degeneracies.
- **A RD model for higher harmonics.**
    - Nonspinning massratio fits.
    - Detectability, Resolvability & Measurability.
- **Conclusions & future work (C&F).**



# State-of-the-art

**· 10 BBH observations during O1 and O2**




**1- Final Mass in [18 , 80]**

**2- Massratios in [1, 9 ?]**

**3- Final spin in [0.67, 0.81]**

**4- Loudest SNR ~ 8.5 at 3ms after h peak**

GW150914

**5- Evidence of subdominant modes**

GW190814 ! ⟶ q~9 !

GW190412 ! ⟶ q~4 !

**6- Overtones?** GW150914 ?

## Characteristic 'chords' of BHs

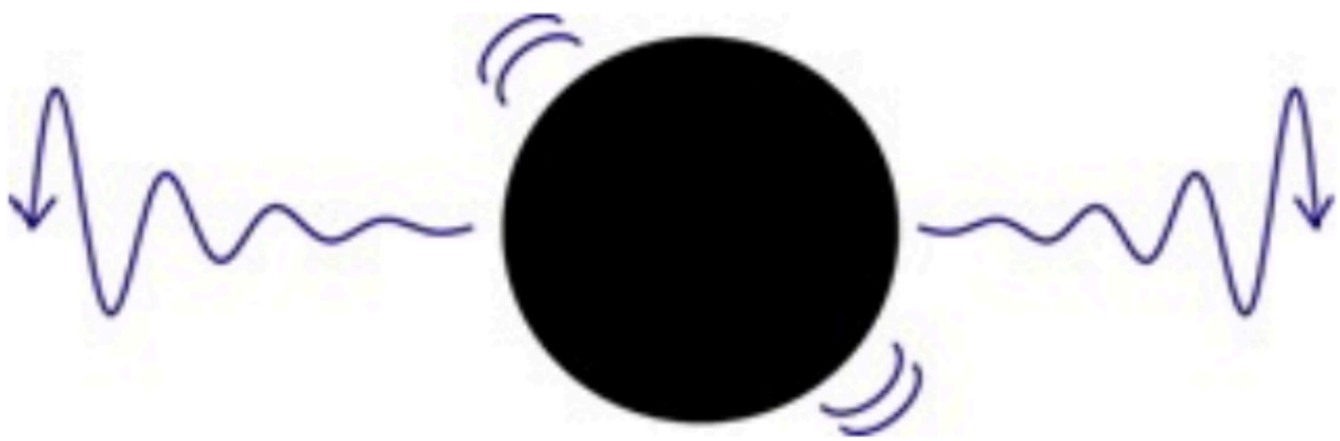


*Image source:* http://slideplayer.com/slide/4179737/

Occurs when the geometry is perturbed by any scalar, vector or tensor perturbation.

$$g_{ab} = g^0_{ab} + h_{ab} \qquad \boxed{|h_{ab}| << |g^0_{ab}|}$$

The perturbation is emitted back as a superposition of dissipative (decaying) modes with characteristic frequencies and damping times.

# On the Ringdown, overtones and BH spectroscopy

- Ringdown Waveform $h = \Sigma_{lmn} \mathcal{C}_{lmn} \boxed{e^{-\iota\omega_{lmn}(t-t^0_{lmn})} e^{-(t-t^0_{lmn})/\tau_{lmn}}} \mathcal{Y}_{lm}$ $\quad C_{lmn} = A_{lmn} e^{\iota\phi_{lmn}}$

Overtones

Set of QNM amplitudes & phase shifts

**<u>The QNM spectra spectrum: unique to the mass & spin of the BH</u>**

Overtones and modes have been know to be important in RD modelling and considered as well for BH spectroscopy

# On the Ringdown, overtones and BH spectroscopy

***~ v/c expansions***

***~ Nonlinear regime***

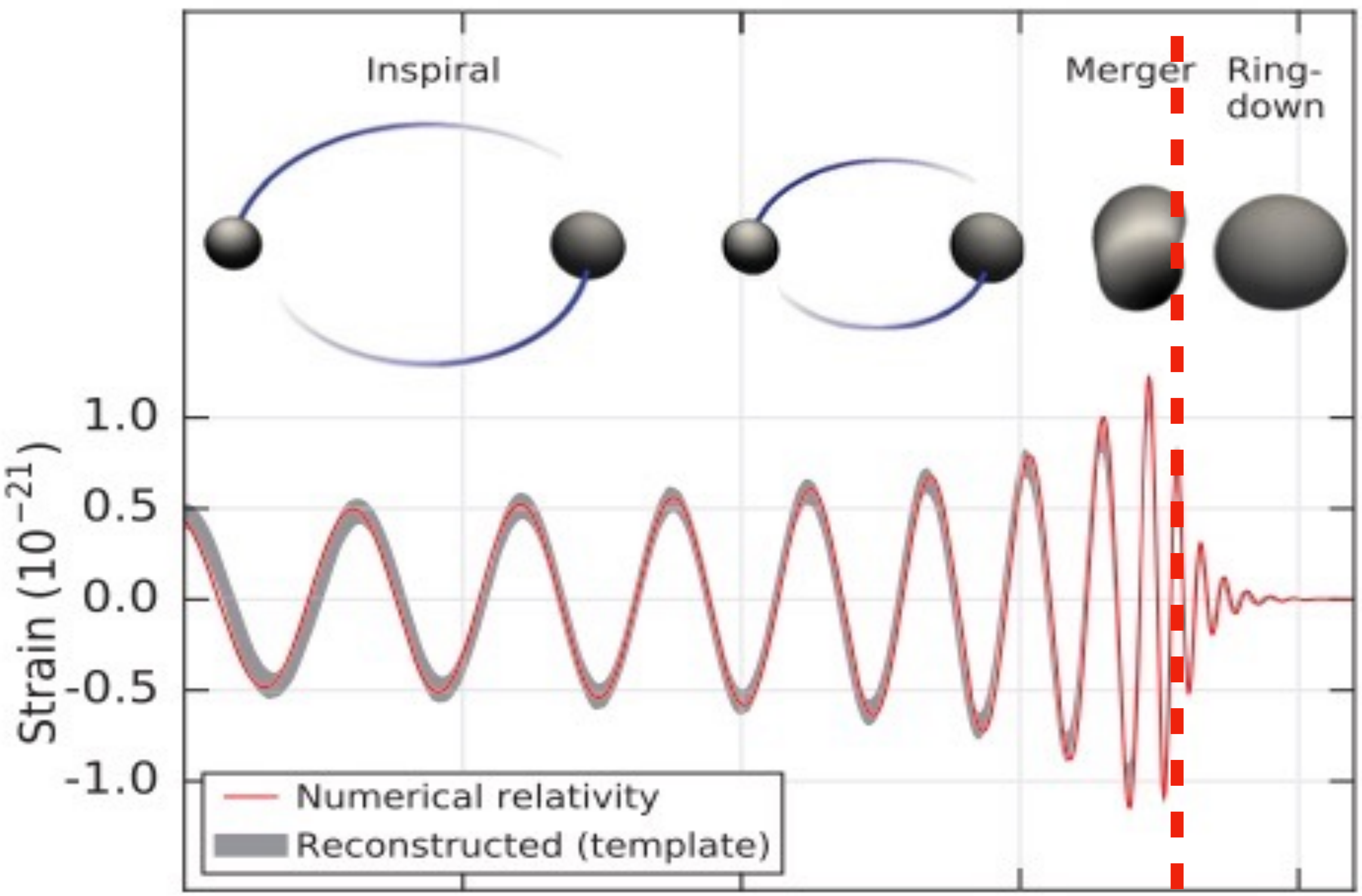
***~ Linear regime***

***('far' from merger)***

***Well described by PT***

Teukolsky, Chandrasekhar, Ferrari,

Kokkotas, Berti, Cardoso...



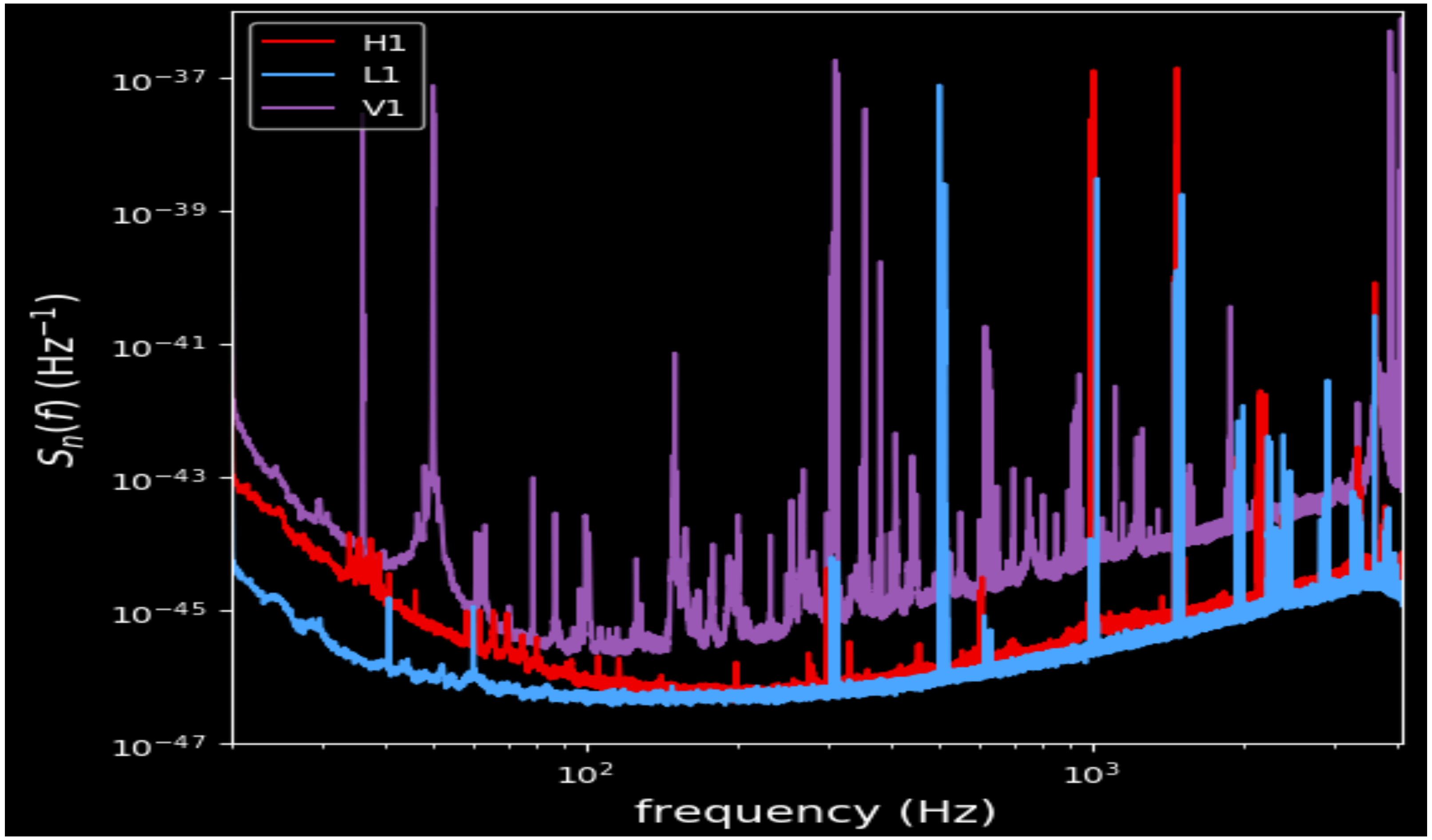

Phys.Rev.Lett. 116 (2016)

Though the exact position is not well known

To estimate the BH parameters we need a model for the likelihood function.

$$log\, p(d|\vec{\theta}) \propto \langle d - h_T | d - h_T \rangle \quad , \quad d = h + n \quad , \quad \rho^2 \sim \langle a|b \rangle = 4\mathbb{R} \int_{f_{low}}^{f_{max}} \frac{a(f)\, b^*(f)}{S_n(f)} df$$

The likelihood function requires both a signal model and a noise model.



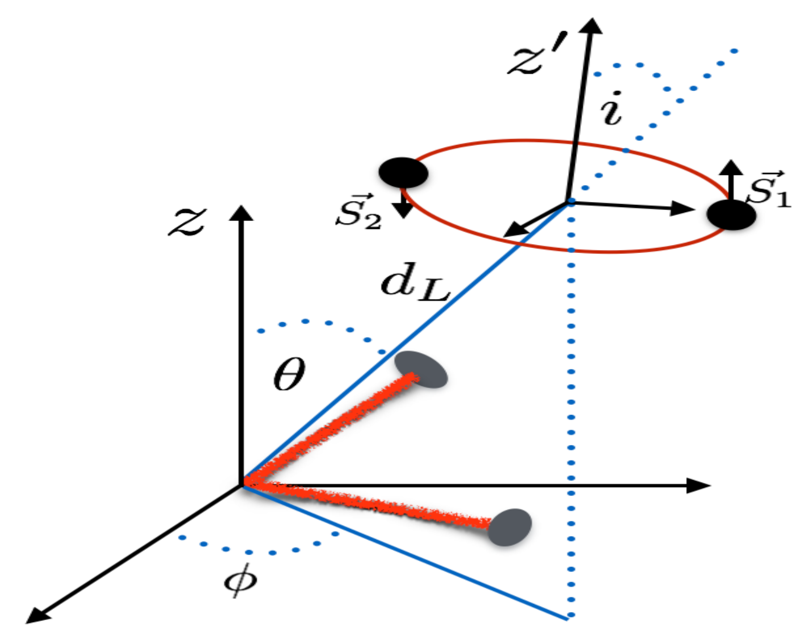
# On the Ringdown, overtones and BH spectroscopy

- Ringdown Waveform $h = \Sigma_{lmn} \mathcal{C}_{lmn} \boxed{e^{-\iota\omega_{lmn}(t-t^0_{lmn})} e^{-(t-t^0_{lmn})/\tau_{lmn}}} \mathcal{Y}_{lm}$ $\quad C_{lmn} = A_{lmn} e^{\iota\phi_{lmn}}$

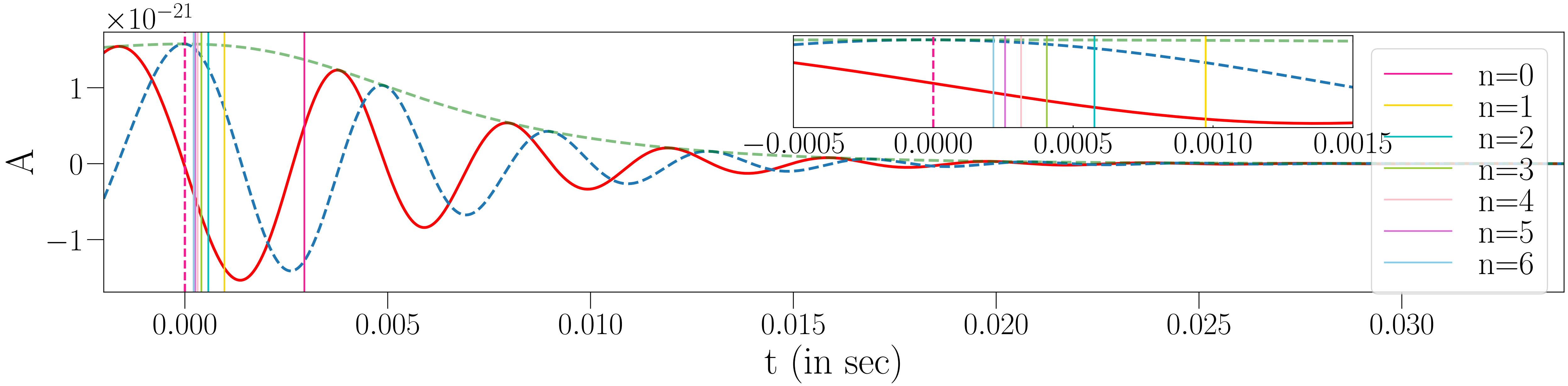
- l=m=2 is the dominant mode. Where most SNR comes in.

- l=m≠2 more excited for
  - Sources not aligned with the line of sight
  - Highly asymmetric sources (high q, high a)

- Same for n tones. Relevance of n↓ as n ↑



# On the Ringdown, overtones and BH spectroscopy




<u>Valid for moderate spins</u>

$$\omega_{lmn}(a_f, M_f) \approx \frac{l}{2}\omega_{22n}(a_f, M_f) \qquad \tau_{lmn}(a_f, M_f) \approx \tau_{22n}(a_f, M_f)$$

Mode-tone frequencies O(5Hz) for a GW150914 event

| $n$ | $f_{22n}$ [Hz] | $\tau_{22n}$ [ms] | $f_{33n}$ [Hz] | $\tau_{33n}$ [ms] |
|---|---|---|---|---|
| 0 | 249.59 | 4.1 | 395.55 | 4.0 |
| 1 | 244.09 | 1.3 | 392.57 | 1.3 |
| 2 | 233.88 | 0.8 | 386.94 | 0.8 |
| 3 | 220.26 | 0.6 | 379.30 | 0.6 |
| 4 | 205.66 | 0.4 | 370.43 | 0.4 |
| 5 | 197.56 | 0.4 | 361.10 | 0.3 |
| 6 | 197.43 | 0.3 | 351.96 | 0.3 |
| 7 | 198.08 | 0.3 | 343.50 | 0.3 |

## Black hole no-hair theorem test — [Israel, Hawking, Carter…]

**A BH spacetime in general relativity (GR) is uniquely defined by at most three parameters: the mass, the angular momentum and the electric charge.**

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$\downarrow$

**Perturbative dynamics parametrizable by its mass and spins**

$\swarrow \quad \downarrow \quad \searrow$

$$\omega_{lmn} = f(M_f, a_f) \qquad \tau_{lmn} = g(M_f, a_f) \qquad C_{lmn} = h(M_f, a_f)$$

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$$\downarrow$$

$$(M_f, a_f)$$

$$\swarrow \qquad \downarrow \qquad \searrow$$

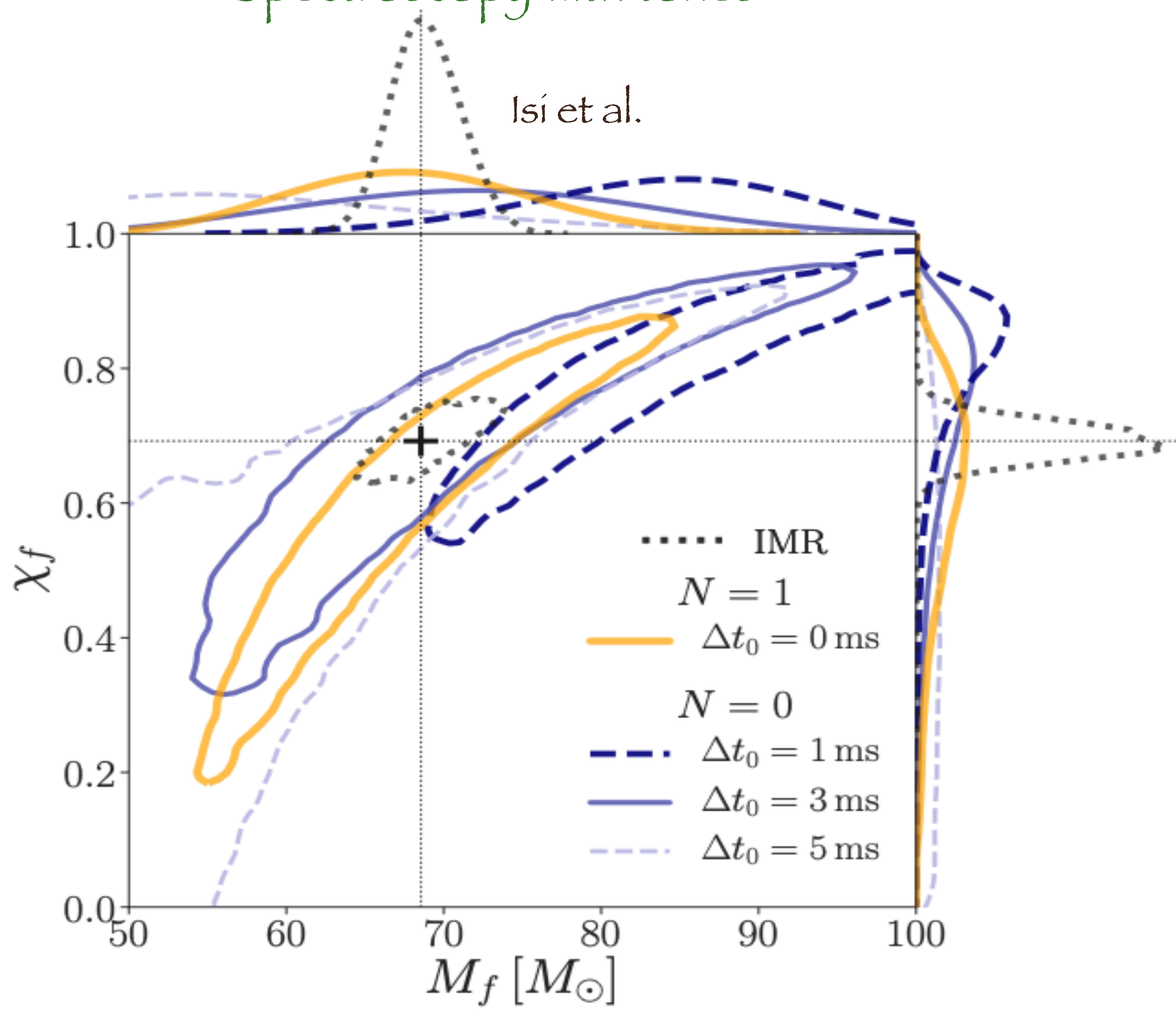
$$\omega_{lmn} \longleftrightarrow \tau_{lmn} \longleftrightarrow C_{lmn}$$

**2- BH spectroscopy or the observation of the QNM spectrum: {An, ϕn, ωn, τn}**

***2.1— An observational validation of the no-hair theorem requires that the QNM frequencies and the damping times must be estimated by performing a Bayesian parameter estimation (PE) on all the RD model parameters (i.e., {An, ϕn, ωn, τn}).***

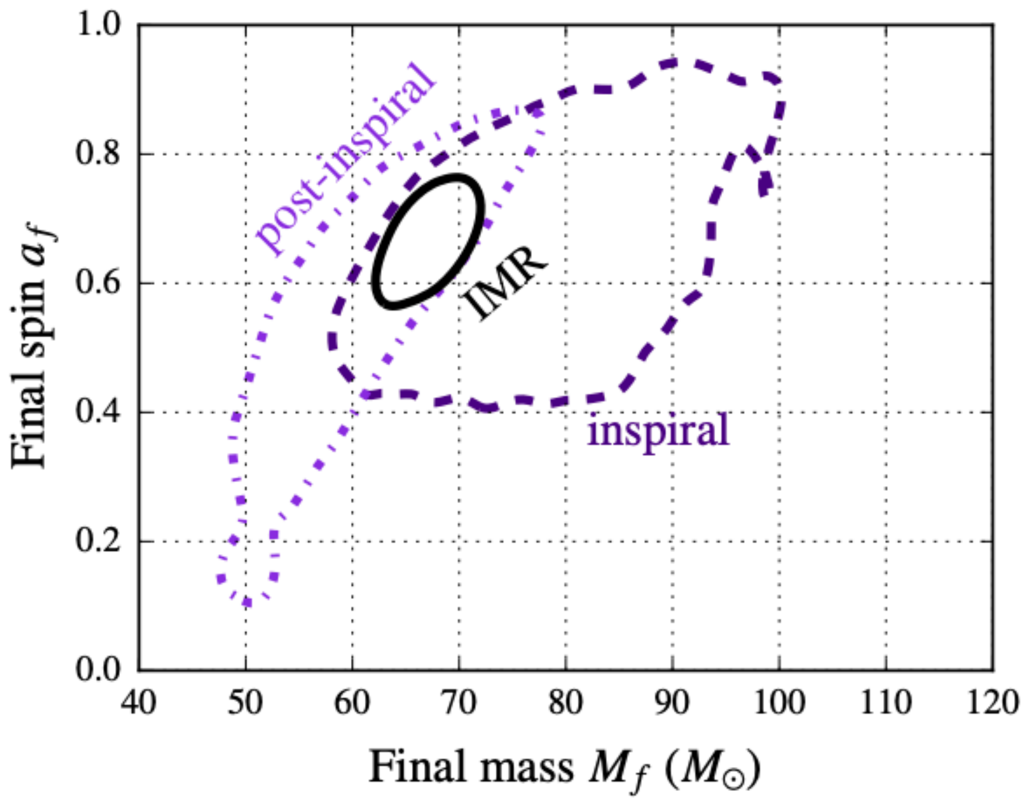
***2.2— It is sufficient to obtain independent measurements of at least two QNM modes.***

Spectroscopy with tones



# Testing the BH no-hair theorem.

**1- IMR consistency tests**




LIGO-Virgo collaboration

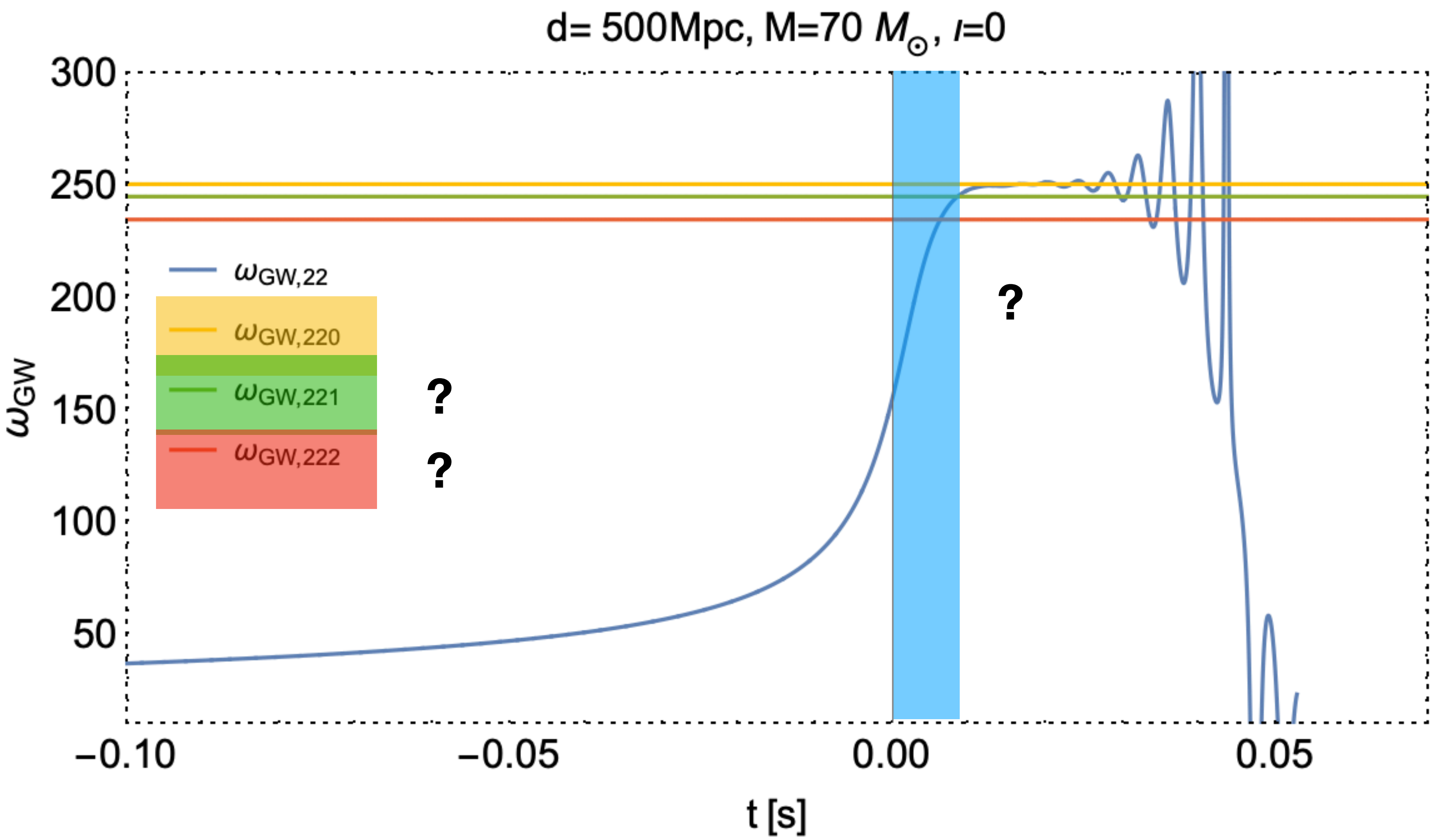
- **Estimate the final mass and final spin from different data chops.**

- **In GR both must agree!**

# An Overtone model for GW150914

Phys. Rev. D 101, 044033 (2020)

# On the Ringdown, overtones and BH spectroscopy

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# On the Ringdown, overtones and BH spectroscopy

Take a NR case compatible with GW150914 (SXS NR Data)

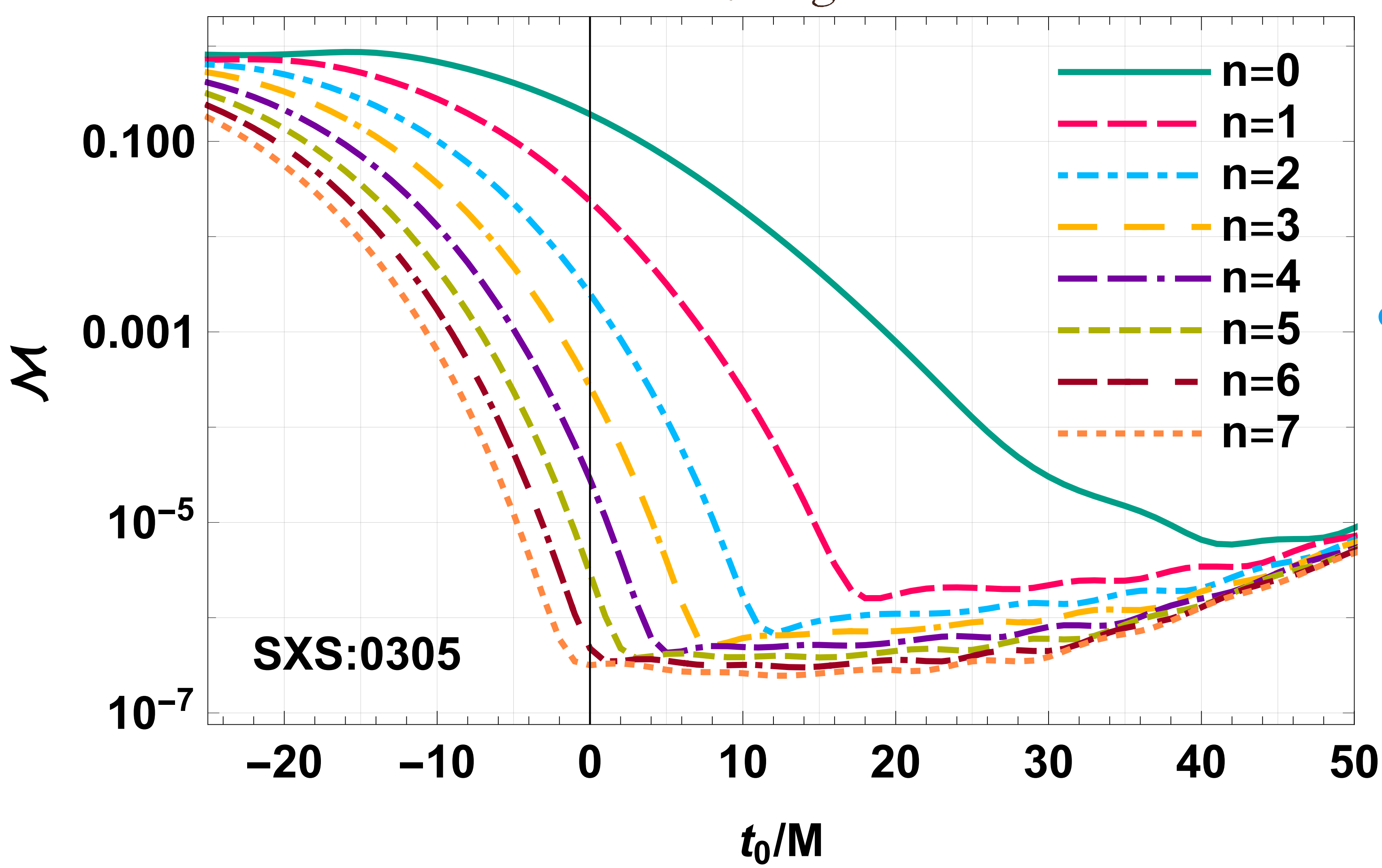
$\longrightarrow$ Fix them to GR QNM spectrum.

$\searrow$ Fit amplitudes $C_{lmn}$ to NR.

London et al., Giesler et al., Bhagwat et al.

$$h = \Sigma_{lmn} \mathcal{C}_{lmn} e^{-\iota\omega_{lmn}(t-t^0_{lmn})} e^{-(t-t^0_{lmn})/\tau_{lmn}}\, \mathcal{Y}_{lm}$$

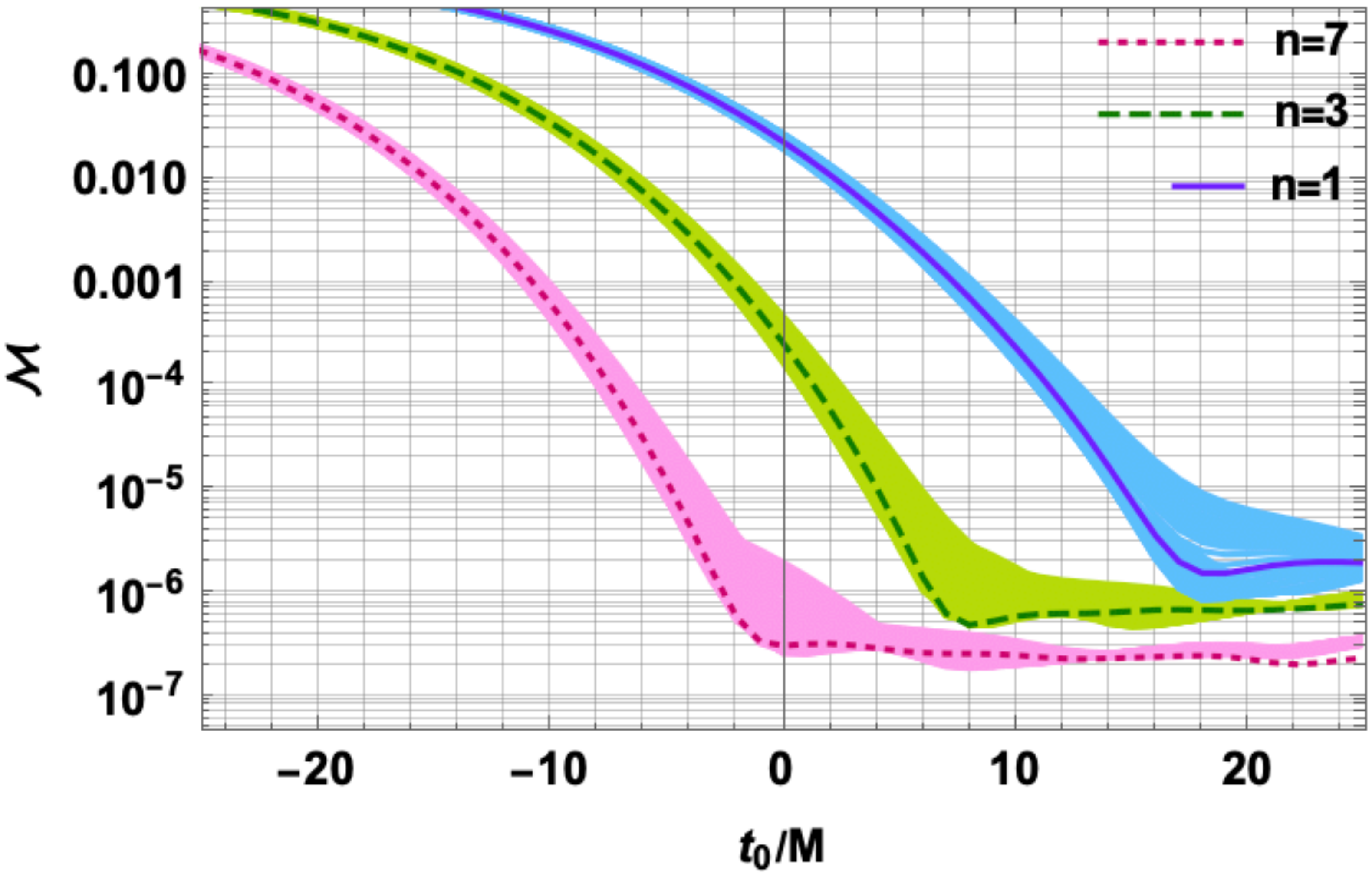
Giesler et al., Bhagwat et al.




Measure of goodness?

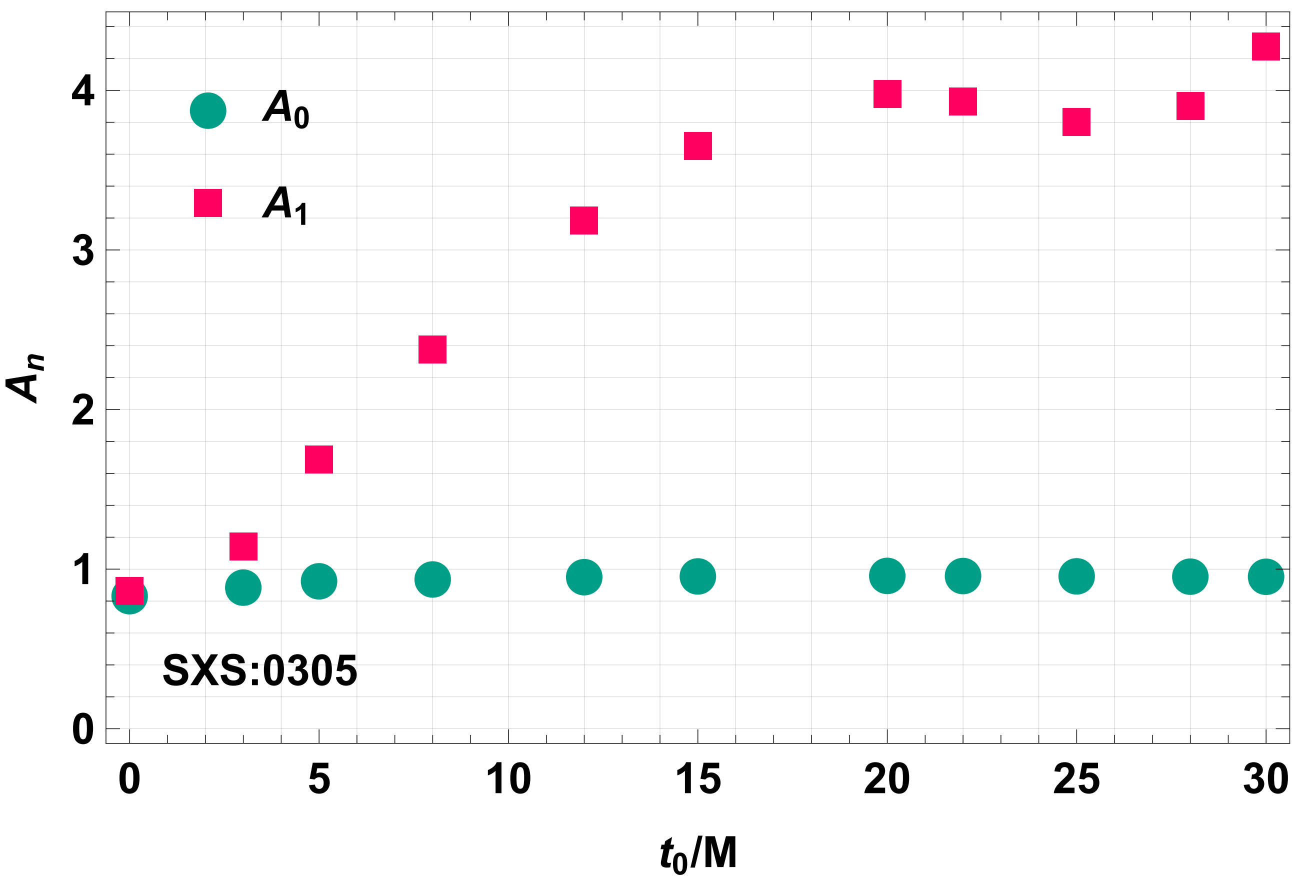
$$\mathcal{M} = \left(1 - \frac{\langle h | h_T \rangle}{\sqrt{\langle h|h\rangle \langle h_T | h_T\rangle}}\right)$$

# Do we really need QNMs to reproduce these fits?

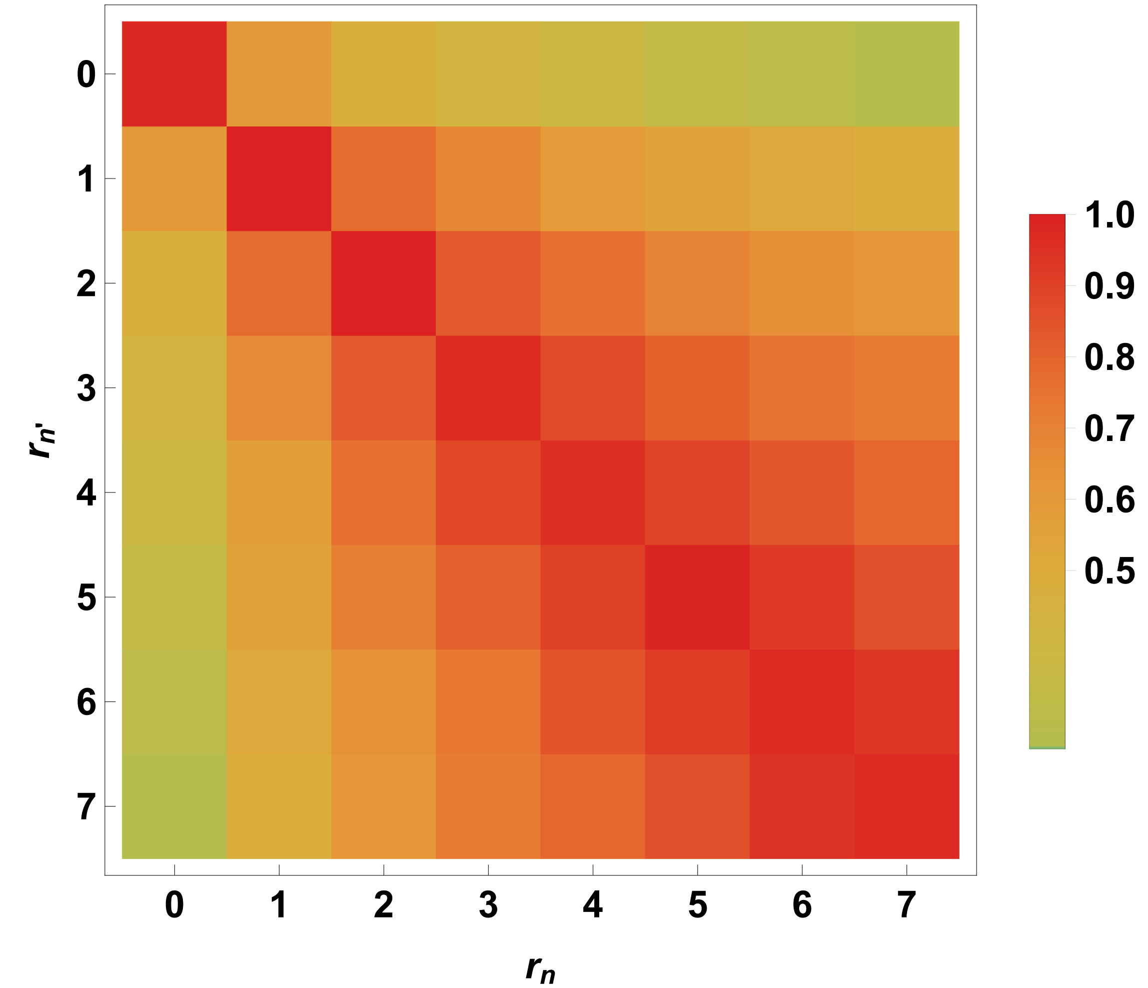



No, so long as we have the frequency of n=0 mode, we can model RD accurately with nonQNM.

# Are we sure that we can calibrate the amplitudes accurately?




Amplitudes wrt. $t_0$




Correlation matrix

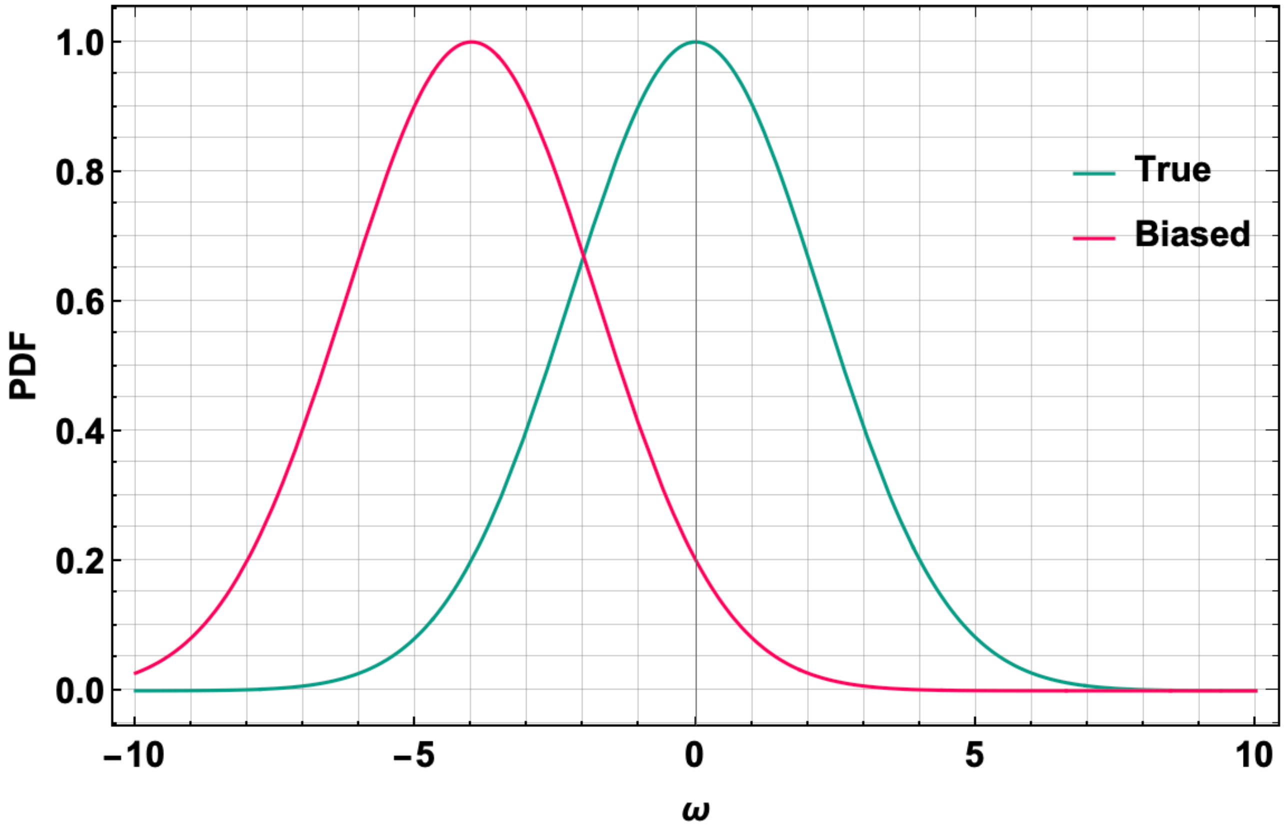
# On the Ringdown, overtones and BH spectroscopy

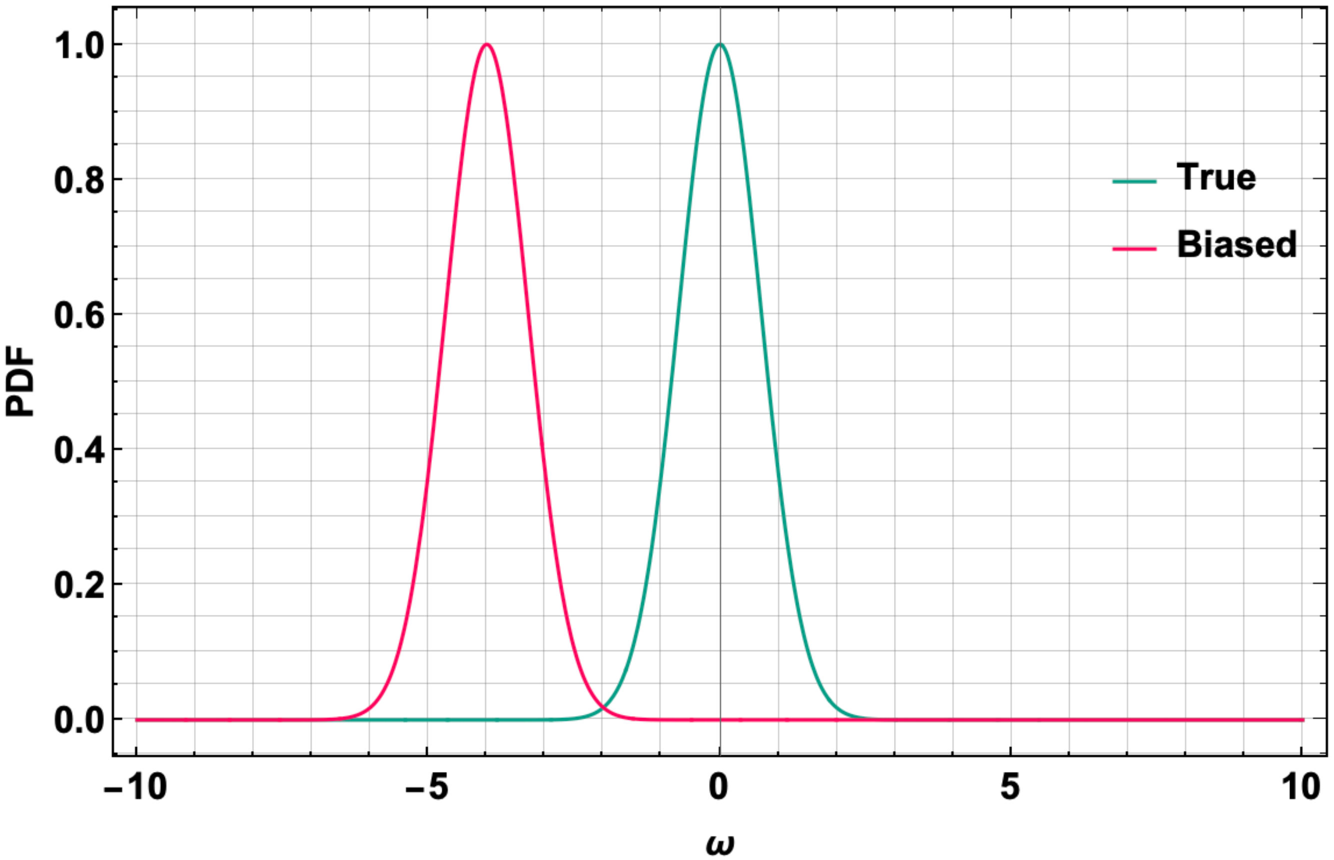
Systematic errors issues , $h \neq h_T \longrightarrow d - h_T \neq n$

Log likelihood function under the assumption of large SNR can be approximated as,

$$log\, p(d|\vec{\theta}) \propto \rho^2 \left(1 - \frac{\langle h|h_T\rangle}{\sqrt{\langle h|h\rangle\langle h_T|h_T\rangle}}\right) \longrightarrow \mathscr{M}$$

Systematic errors $\boxed{\rho \propto \sigma^{-1}}$

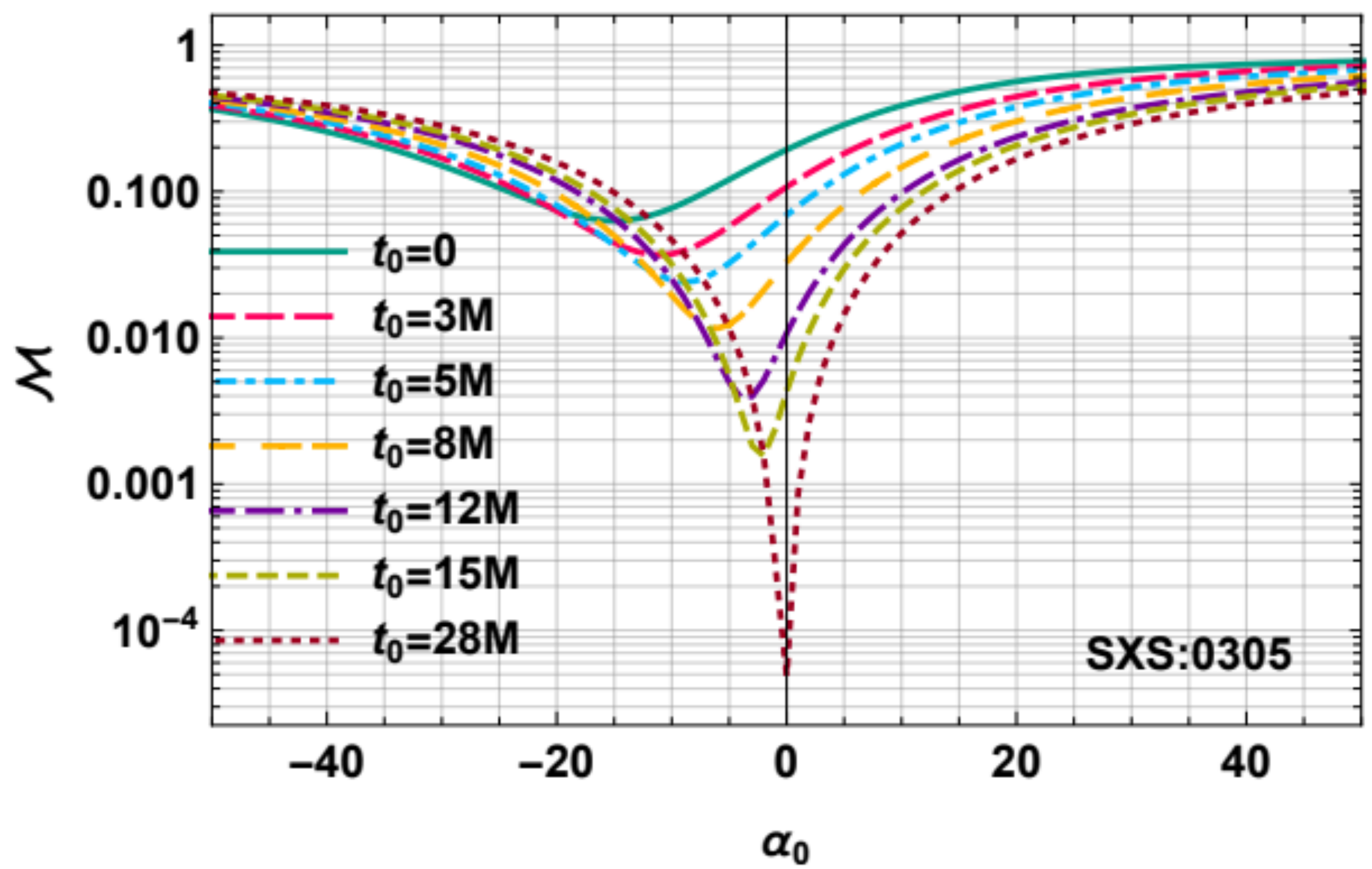


What happens if modify the QNM (again)?

$$h = \Sigma_n A_n e^{-\iota t \omega_n (1+\frac{\alpha_n}{100})} e^{-t/\tau_n}$$

**GW150914** ⟷ **SXS:0305**



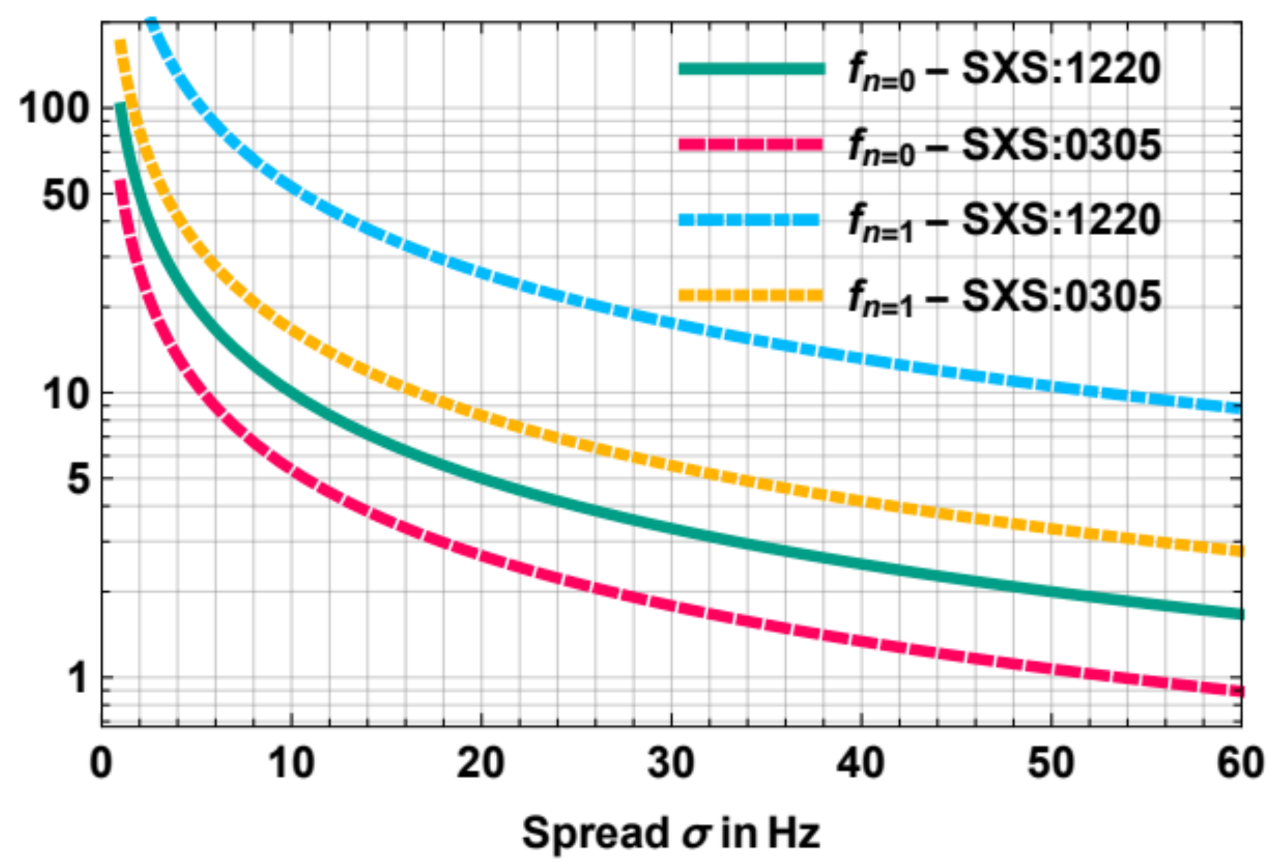
# On the Ringdown, overtones and BH spectroscopy

From Fisher Matrix analysis ——>
$$log\, p(d|\vec{\theta}) \propto \rho^2 (1 - \frac{\langle h|h_T\rangle}{\sqrt{\langle h|h\rangle\langle h_T|h_T\rangle}})$$

- **Resolving tone frequencies is difficult. How difficult?**

Berti et al. ⟶ $\rho > \rho_{\rm crit} = \dfrac{\mathrm{Max}[\rho\sigma_{f_0}, \rho\sigma_{f_1}]}{|f_0 - f_1|}$

**GW150914** ⟷ **SXS:0305**




**(SXS:1220 -> q=4)**

# Higher modes?

https://arxiv.org/pdf/2005.03260.pdf

$$h = \Sigma_{lmn} \mathcal{C}_{lmn} e^{-\iota\omega_{lmn}(t-t^0_{lmn})} e^{-(t-t^0_{lmn})/\tau_{lmn}} \mathcal{Y}_{lm} \quad C_{lmn} = A_{lmn} e^{\iota\phi_{lmn}}$$

# On the Ringdown, overtones and BH spectroscopy

**- What is the optimal set of QNMs to perform 2-mode/tone BH spectroscopy?, i.e., the $l = 2$, $m = 2$, $n = 0$ mode along with either:**

**a) its first overtone, i.e., the $l = 2$, $m = 2$, $n = 1$**

**b) another angular mode, i.e, either the $l = 3$, $m = 3$, $n = 0$ or the $l = 2$, $m = 1$, $n = 0$**

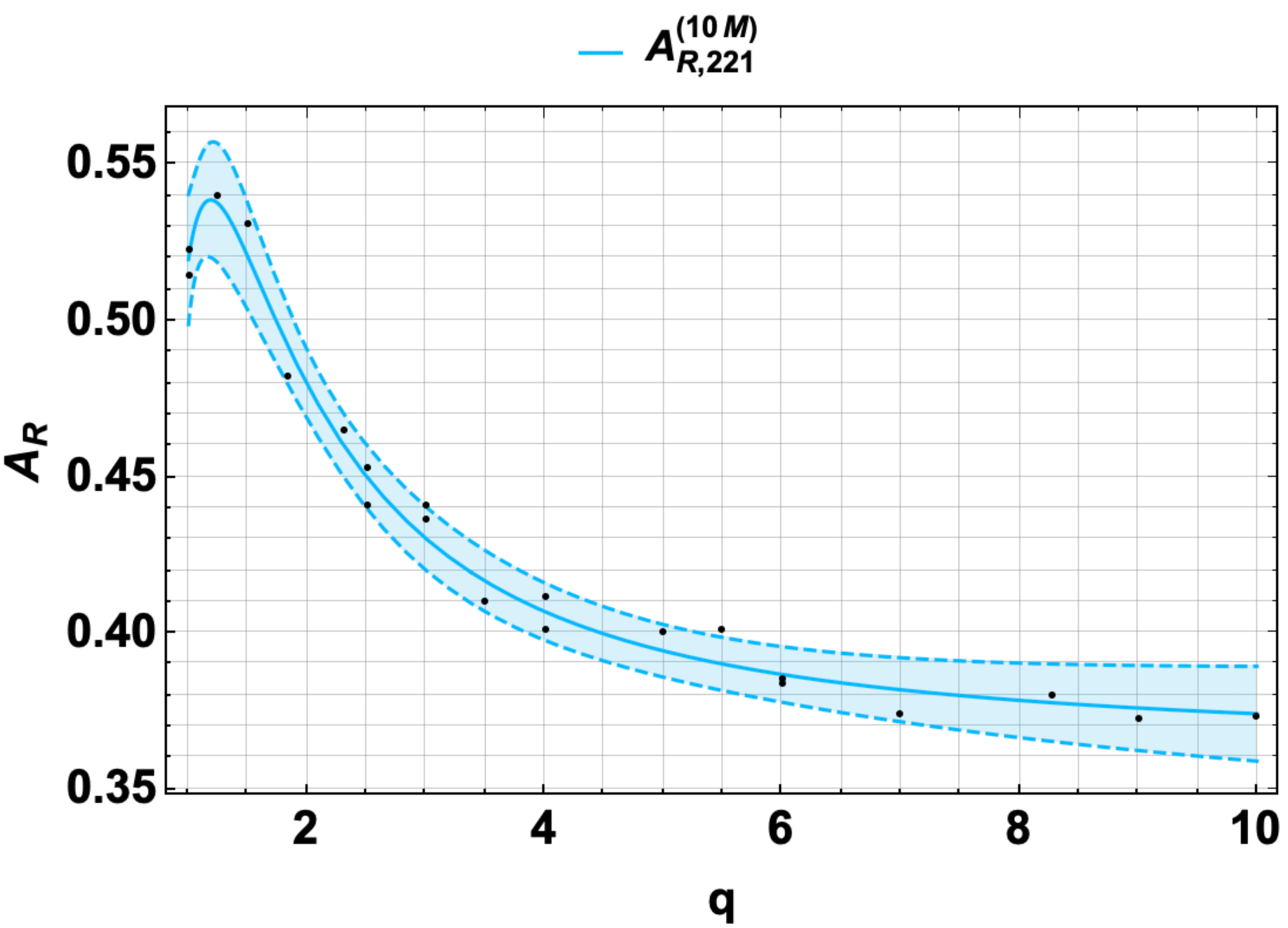
**- What is the optimal set of QNMs to perform 2-mode/tone BH spectroscopy which requires less SNR?**

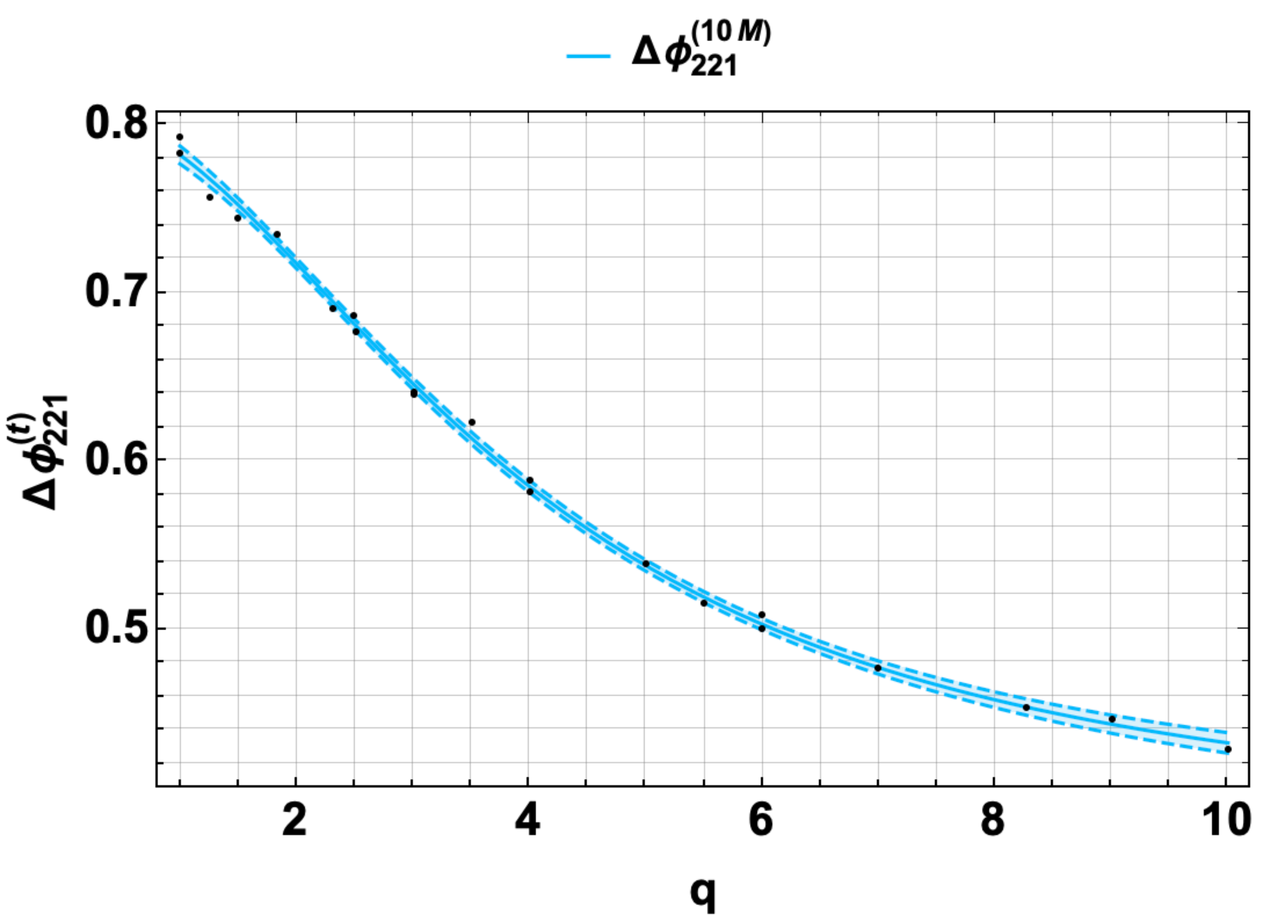
**- Which physical configuration is potentially more promising for detection (if any)?**

# We need models for the amps. and phase shifts!

**SXS catalogue** $\longrightarrow$ $C_{lmn} = A_{lmn} e^{\iota\, \phi_{lmn}}$

**Fits to NR to have my template complete: tones**



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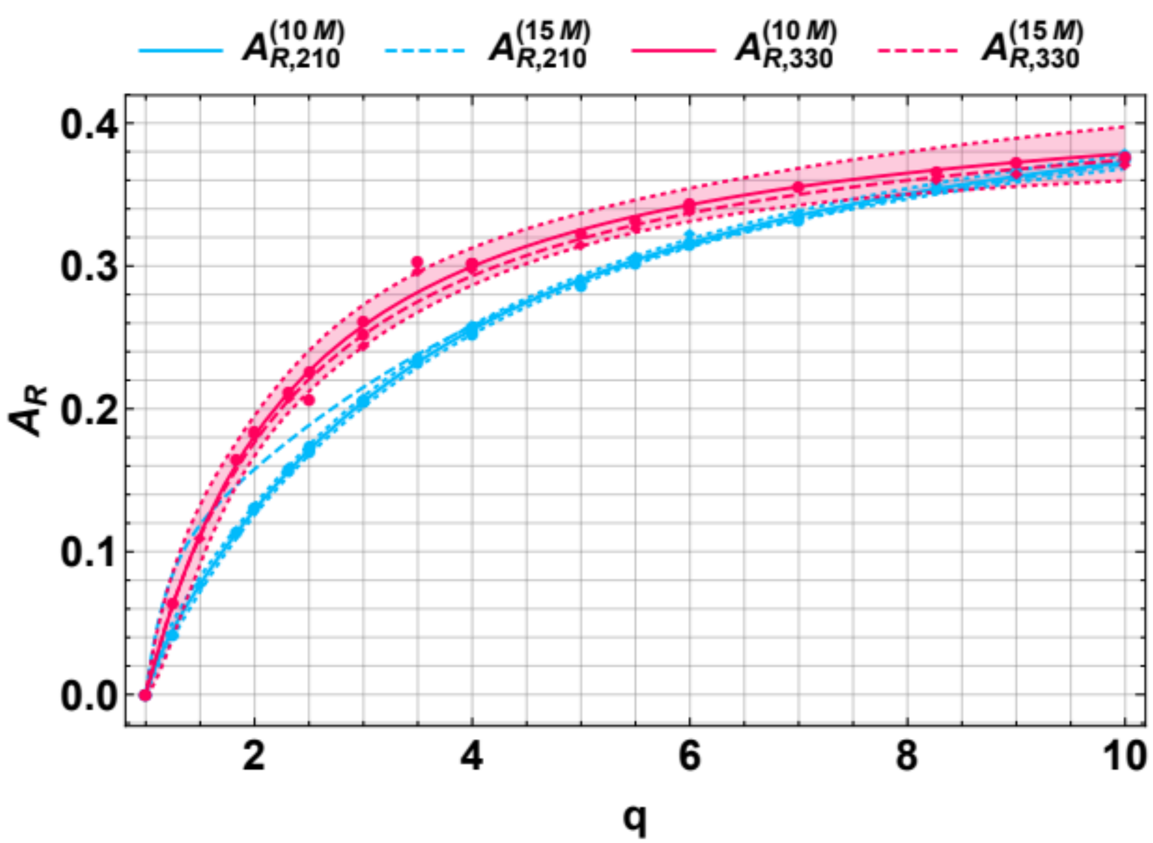
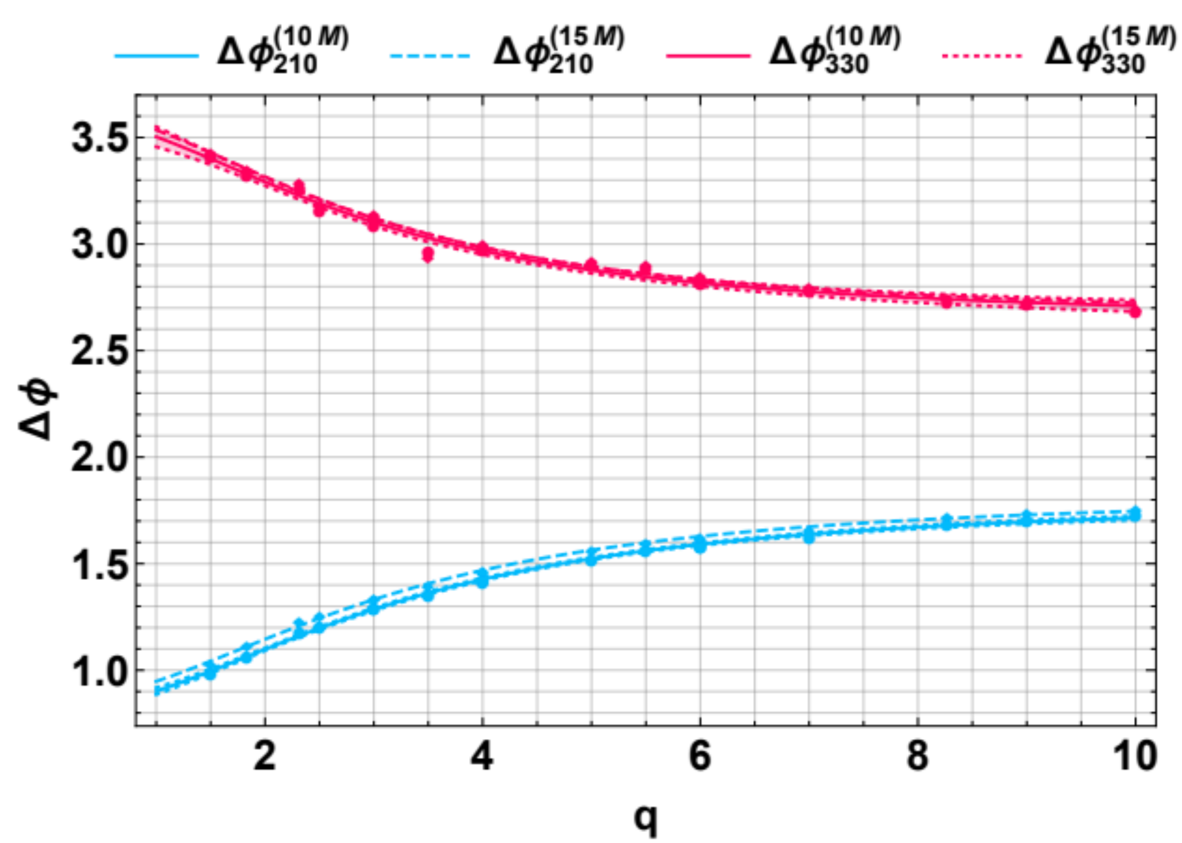
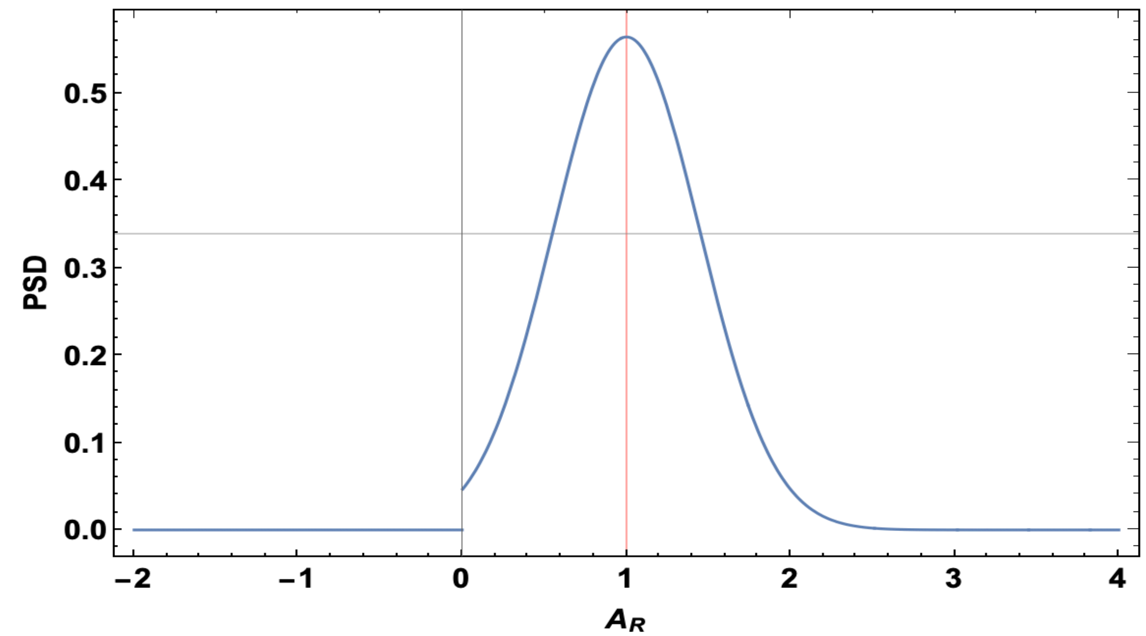


## We require for black hole spectroscopy:

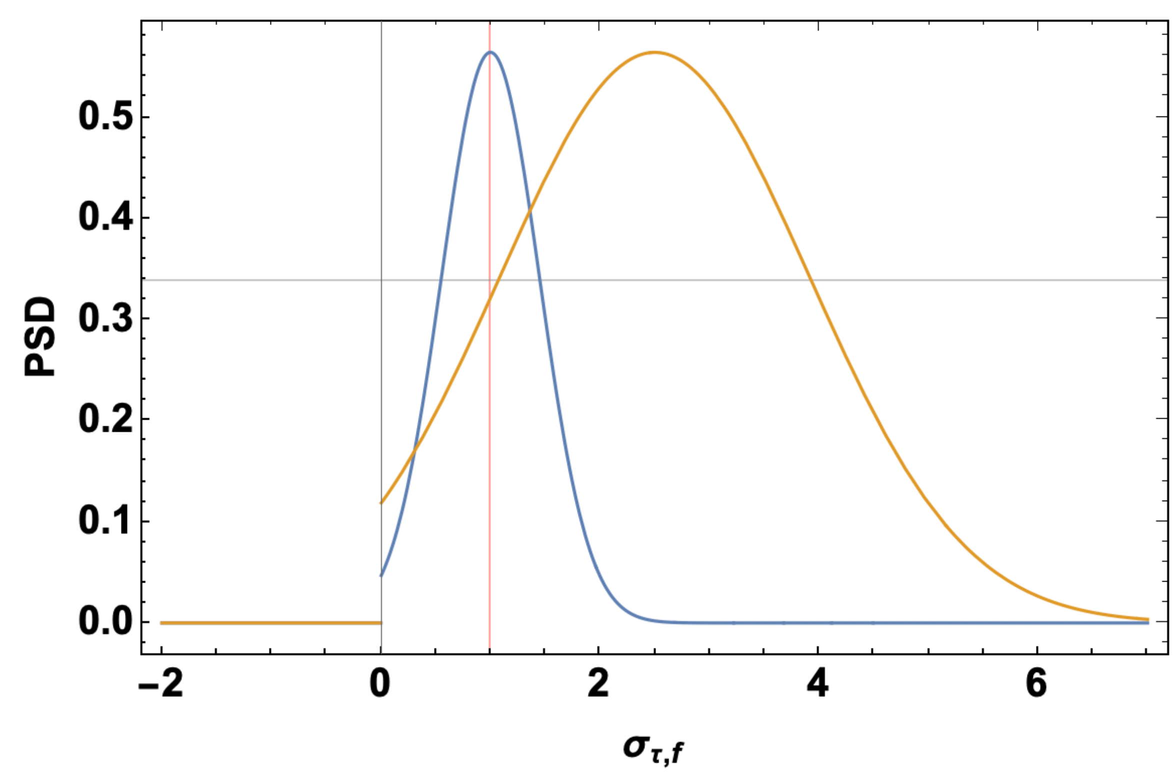
1. *Detectability criterion:*

$$\sigma_{A_R} < A_R$$




2. *Resolvability criterion:* $Q = \pi \tau f$

$$\max[\sigma_{f_{220}}, \sigma_{f_{\rm sub}}] < |f_{220} - f_{\rm sub}|\,,$$
$$\max[\sigma_{Q_{220}}, \sigma_{Q_{\rm sub}}] < |Q_{220} - Q_{\rm sub}|\,,$$




3. *Measurability criterion:*

$$\left\{\frac{\sigma_{f_{220}}}{f_{220}}, \frac{\sigma_{Q_{220}}}{Q_{220}}, \frac{\sigma_{f_{\rm sub}}}{f_{\rm sub}}\right\} \leq T\,,$$
$$\left\{\frac{\sigma_{f_{220}}}{f_{220}}, \frac{\sigma_{Q_{220}}}{Q_{220}}, \frac{\sigma_{Q_{\rm sub}}}{Q_{\rm sub}}\right\} \leq T\,,$$

# 4. Are requirements fulfilled?

1. *Detectability criterion:*

$$\sigma_{A_R} < A_R$$

2. *Resolvability criterion:*

$$\max[\sigma_{f_{220}}, \sigma_{f_{\rm sub}}] < |f_{220} - f_{\rm sub}|\,,$$
$$\max[\sigma_{Q_{220}}, \sigma_{Q_{\rm sub}}] < |Q_{220} - Q_{\rm sub}|\,,$$

3. *Measurability criterion:*

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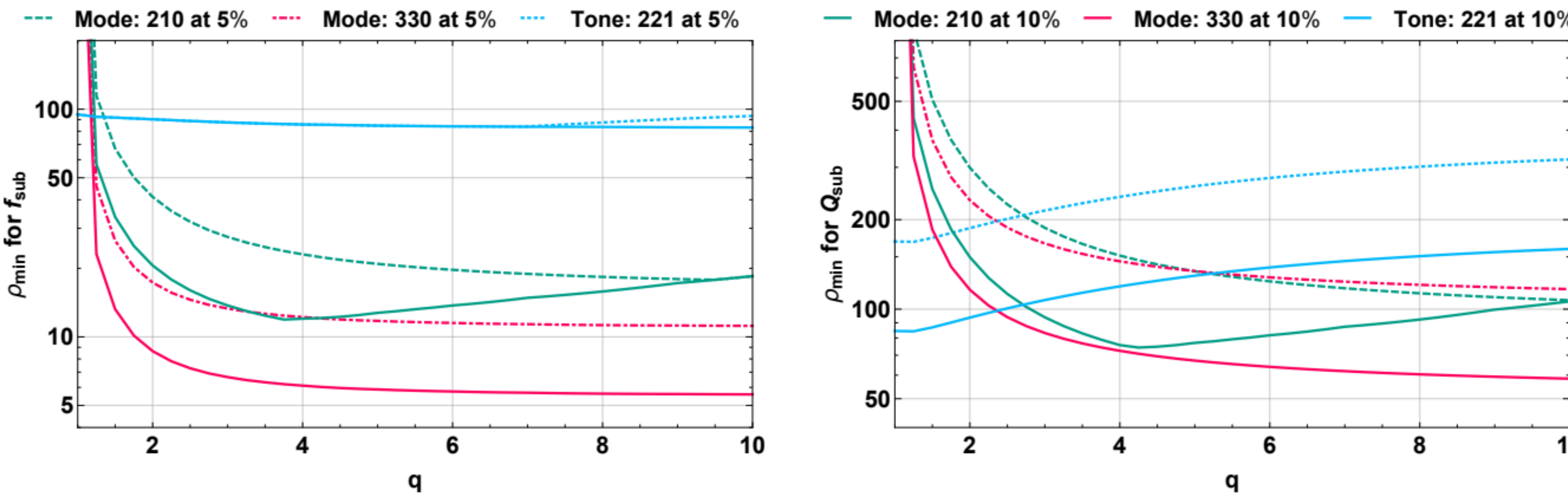



GW190814 ! ⟶ q~9 —> snr into the 33 ~ 6

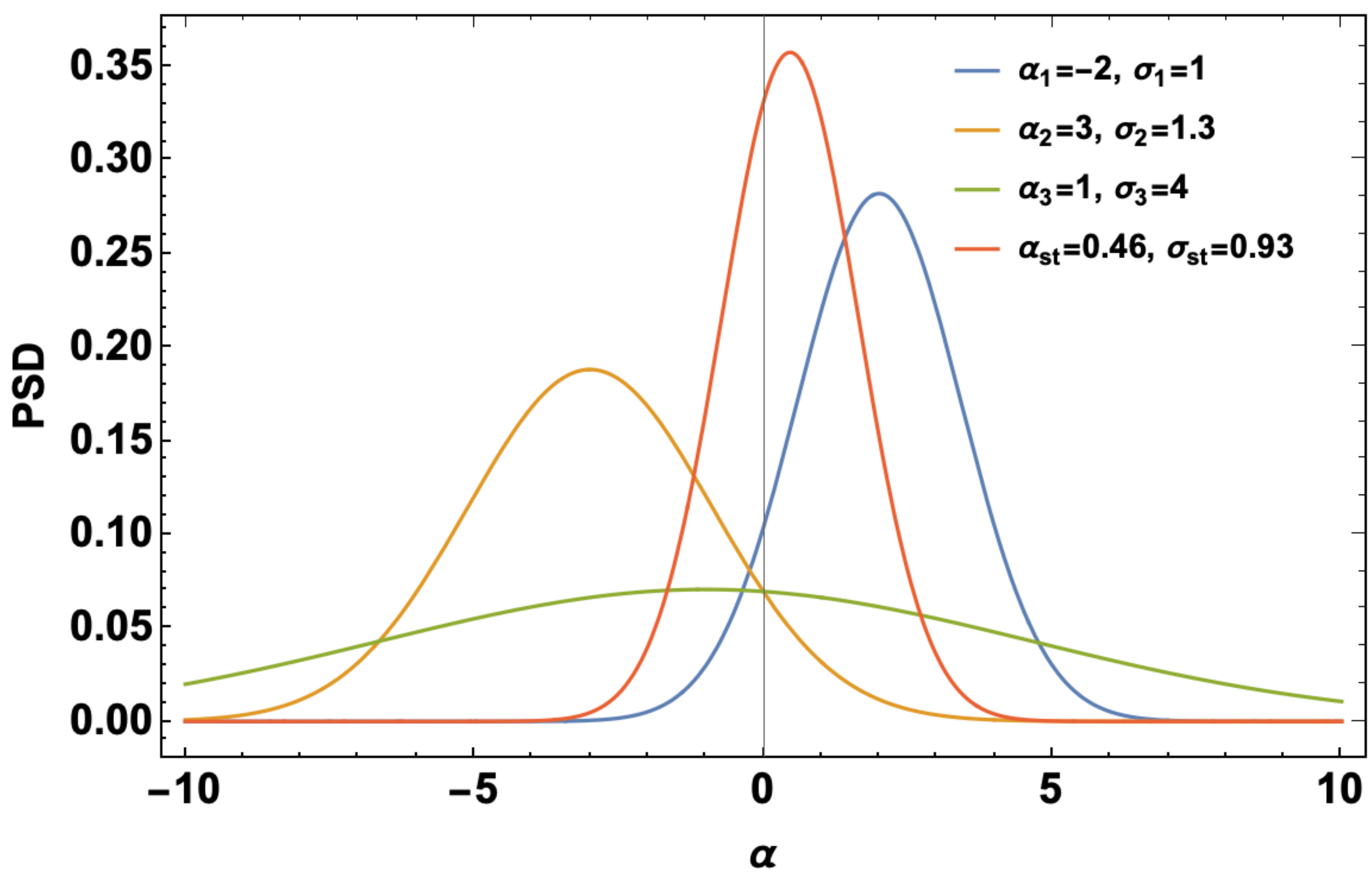
# What does stacking means?

**Probability of observing N independent events:** $P(\alpha) = \Pi_i^N P_i(\alpha_i) \sim \Pi_i^N e^{\frac{(\alpha-\alpha_i)^2}{2\sigma_i^2}}$

**Gaussian noise**

**Simplest scenario : $\alpha_i$ =0** $\longrightarrow$ $P(\alpha) \propto e^{\frac{\alpha^2}{2\sigma^2}}$ $\longrightarrow$ $\sigma_N = \frac{\Pi_l^N \sigma_l}{\sum_j^N \Pi_{l\neq j}^N \sigma_l}$

**α =0.46 , σ = 0.93**




**Stacking of freqs.**

+Carullo
+Cabero...

**Stacking of freqs.**
**+ amps & phases?**

# Conclusions

1. A 8-tone RD model faithfully reproduces a NR -RD from the peak of the amplitude but it is unclear if the damped sinusoids can be interpreted as QNMs.

2. Do the overtones all start simultaneously? Or is this a very strong assumption that might lead to misleading results?

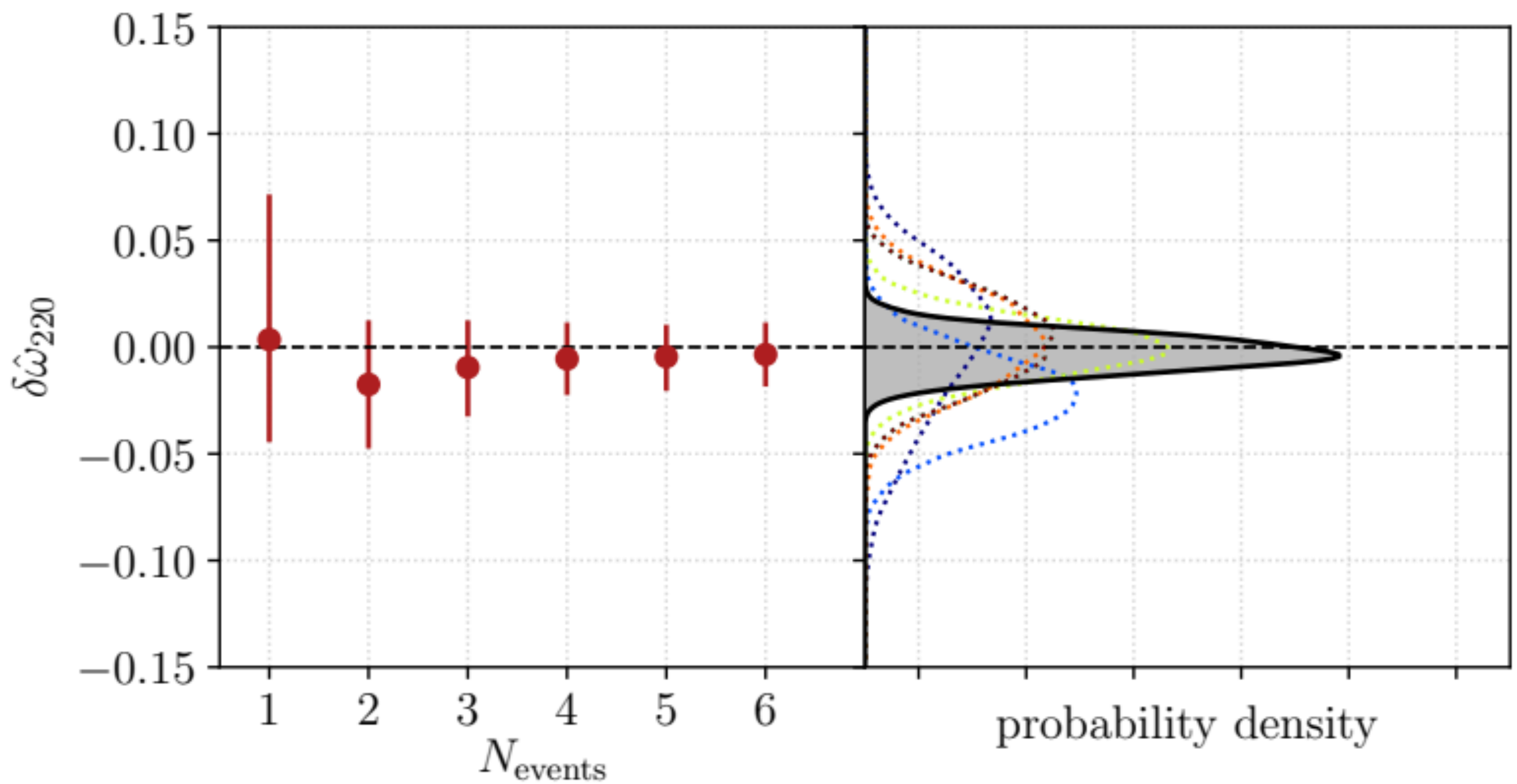
3. For nonspinning binaries with q . 1.2, BH spectroscopy with overtones becomes more convenient, although it requires very loud signals ($\rho$RD & 100).

4. BH spectroscopy could be successfully performed with the $l = m = 3$ mode and at 10% accuracy level if a signal with $\rho$RD ≈ 8 and q ~ 2 is observed.

# BONUS SLIDES

# What does stacking means?

- **N: NR simulations at SNR~ 100, q<3 (or af<~0.7) and Mf~70M**

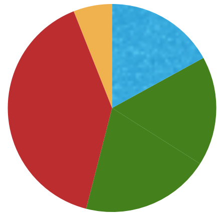


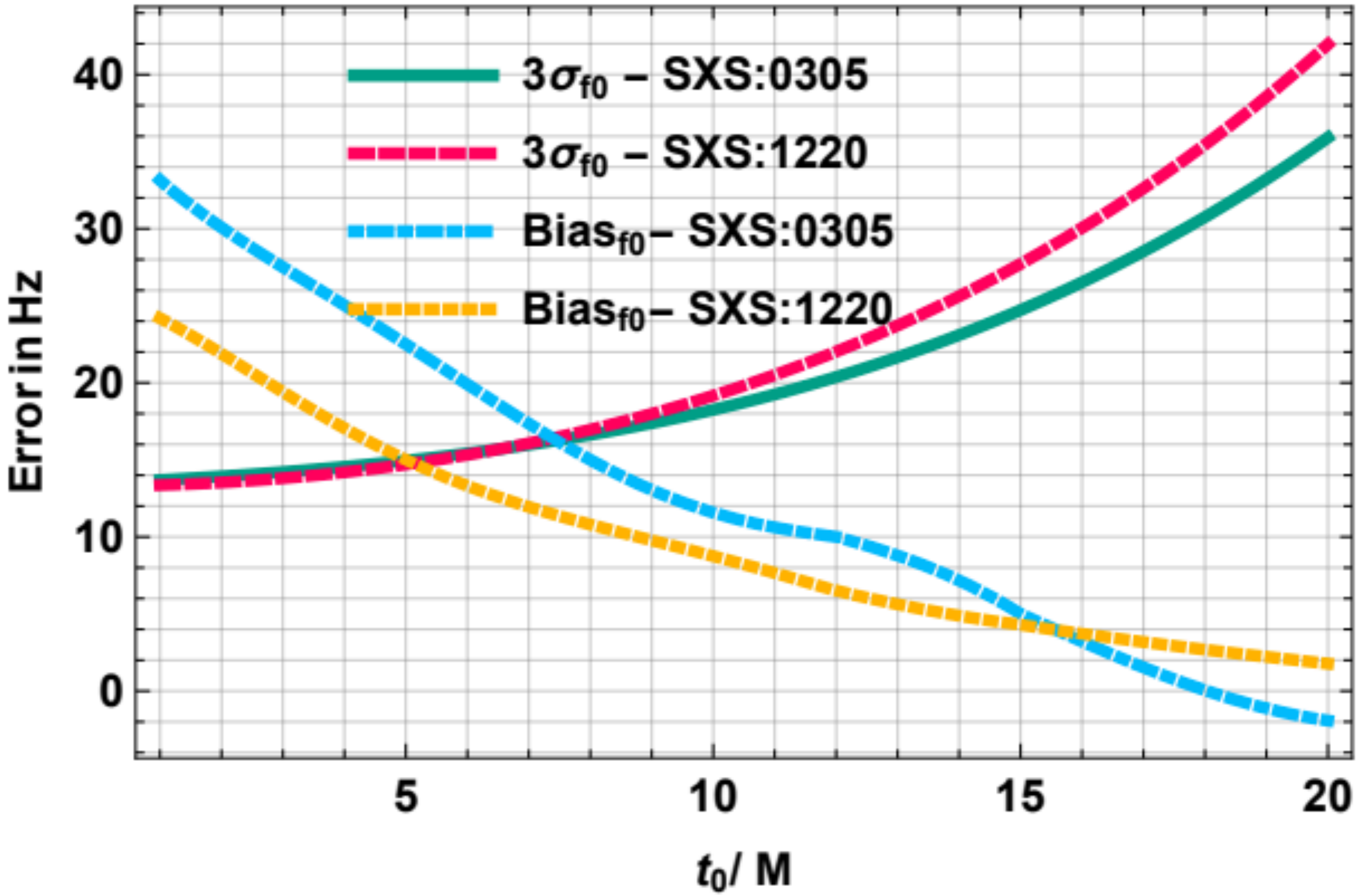

Carullo et al. 2018 (Phys. Rev. D 98, 104020 (2018)

- **With 6 events —> α ~ 1.5% accuracy**

- **Not bad for spectroscopy since w220-w221~ 2.3% for q~1 and Mf~70 .**

# Picking the start time: 2-tone model





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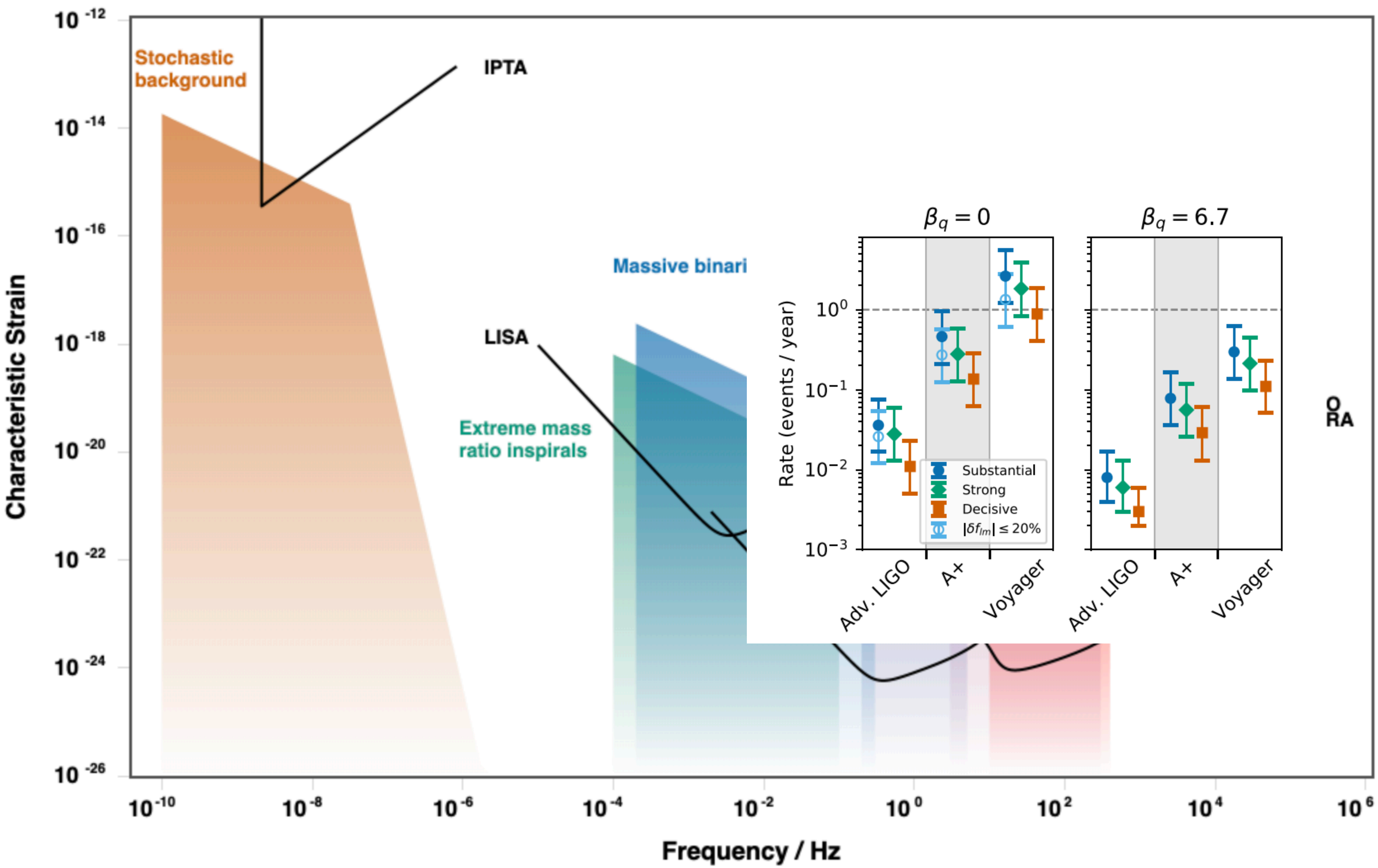

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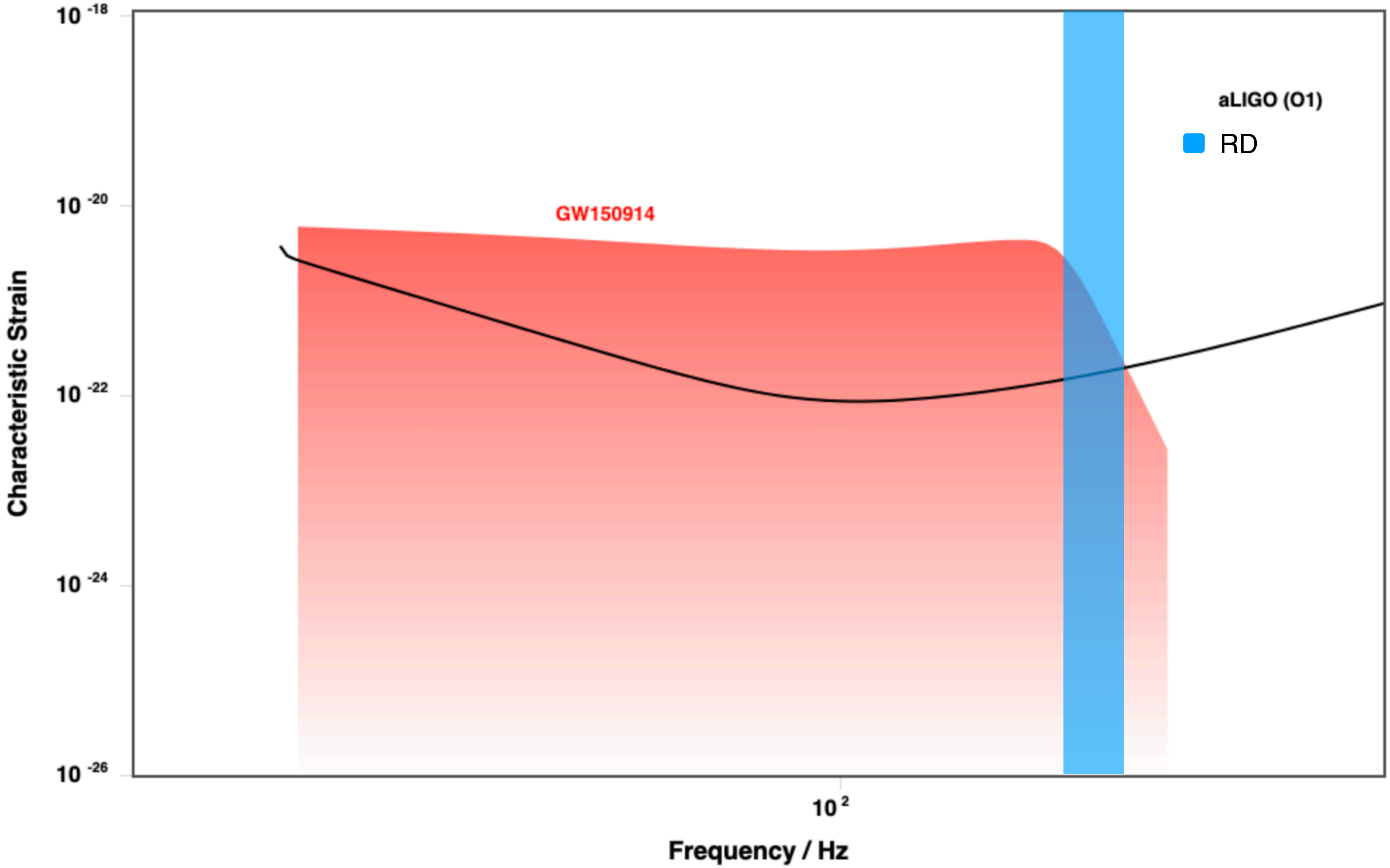
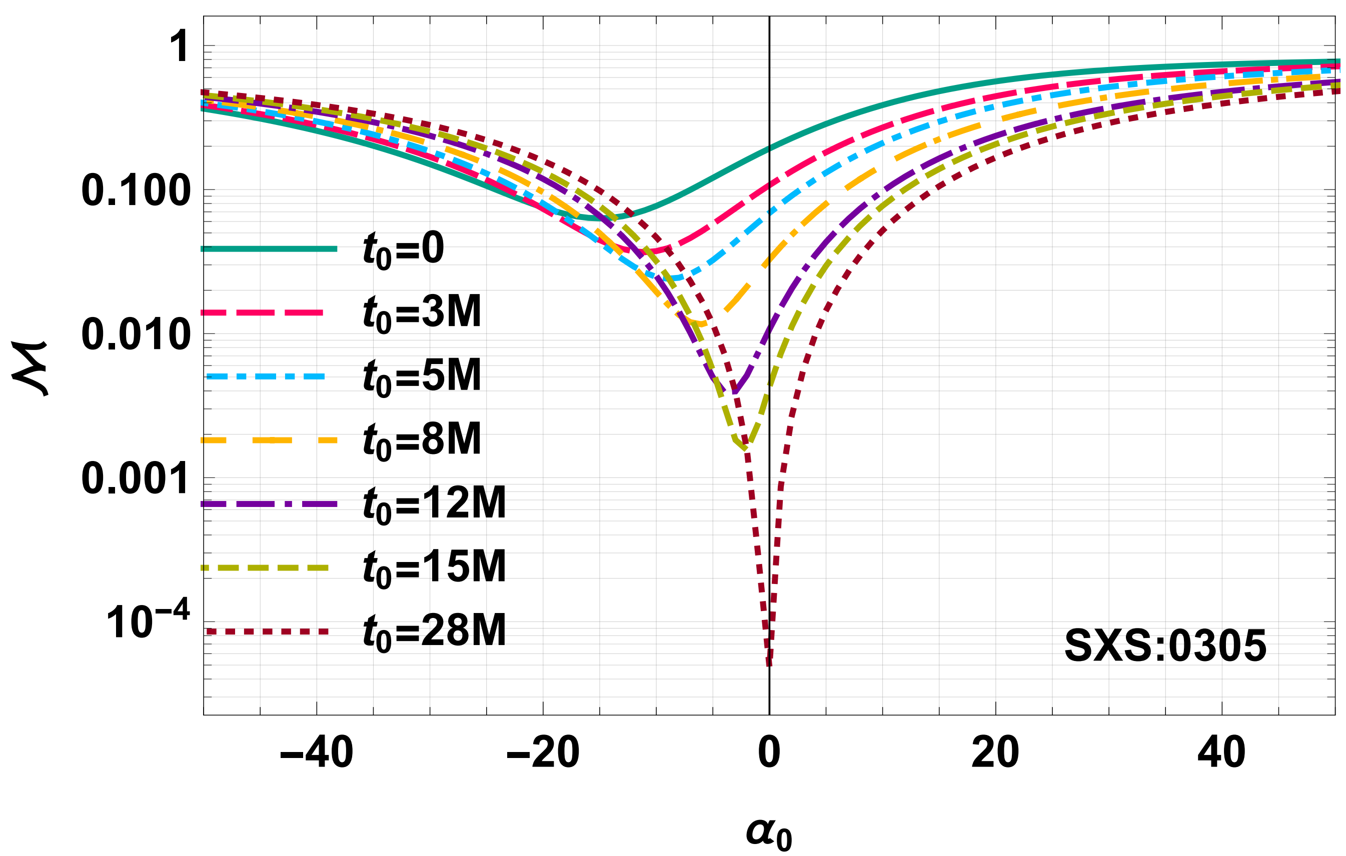


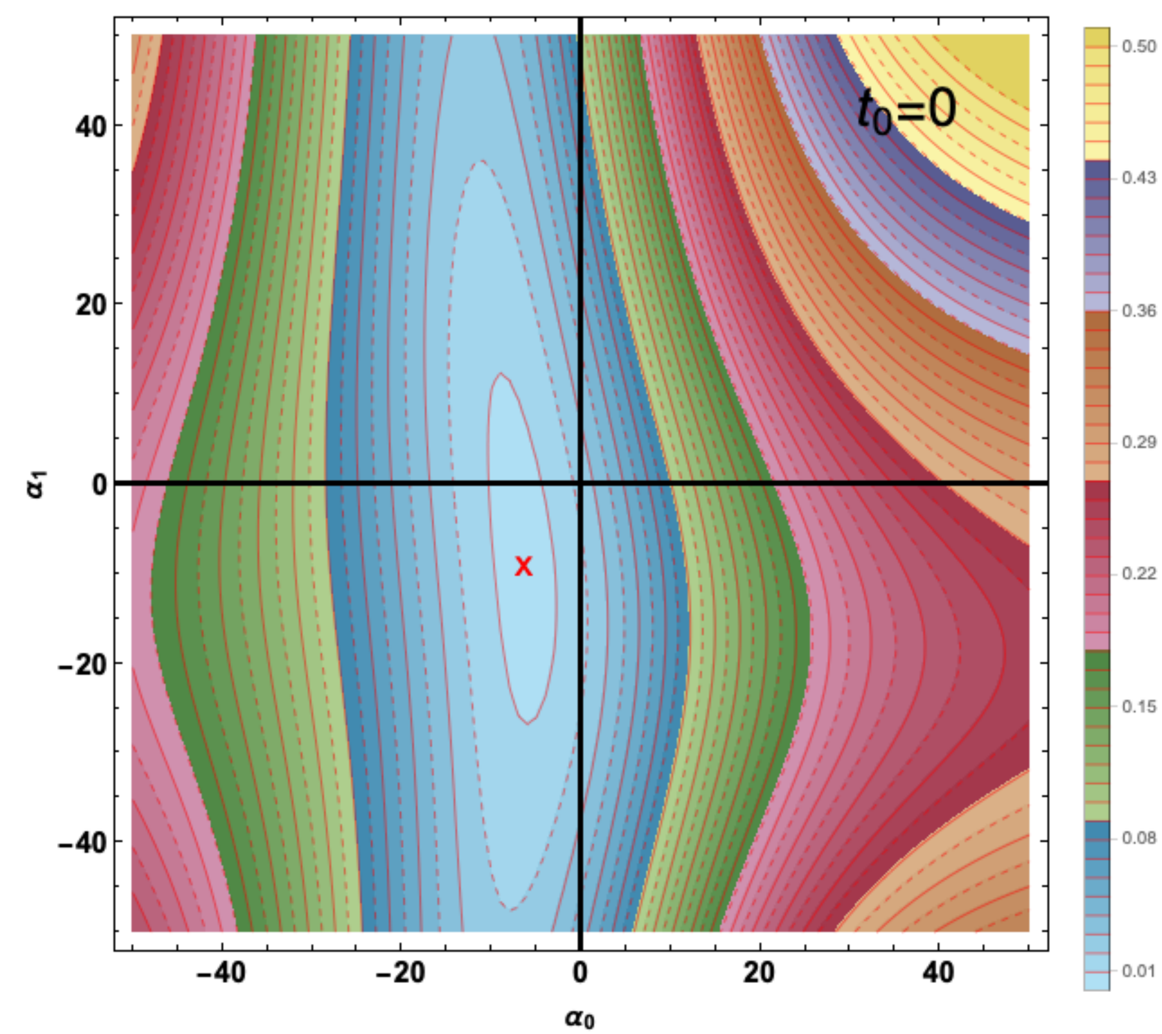

http://gwplotter.com/

# The troubles with modelling RD with overtones

1. We recover the frequencies and damping time all QNMs from a Bayesian PE performed over ALL the RD model parameter.

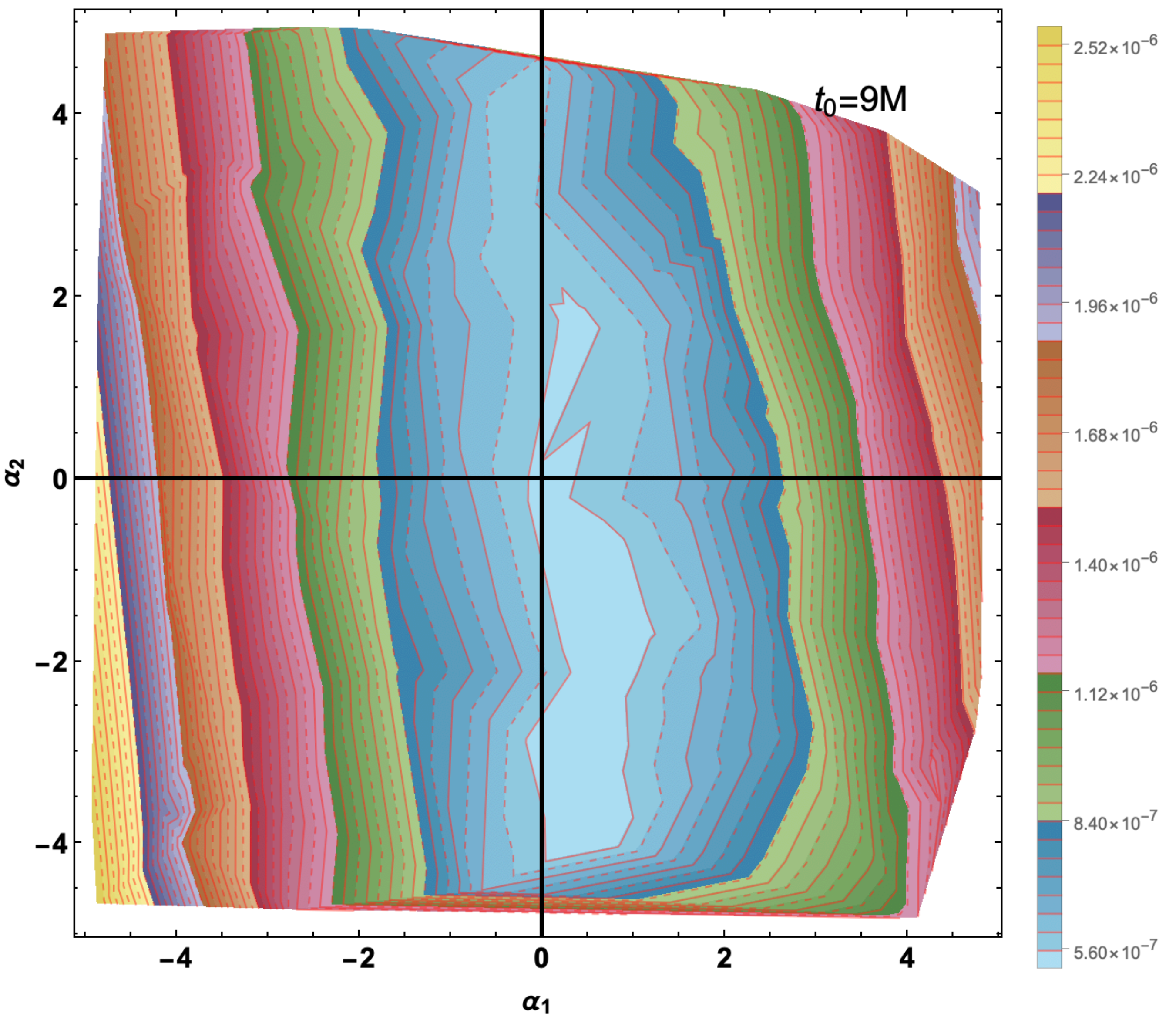
$$h = \sum_n C_n e^{\iota(t-t^0)\omega_n(1+\frac{\alpha_n}{100})} e^{-(t-t^0)/\tau_n}$$




The difference $|\mathcal{M}(\alpha_n) - \mathcal{M}(\alpha_n = 0)|$ implies a systematic Bias.

# 2. Do we really need QNMs to reproduce these fits?



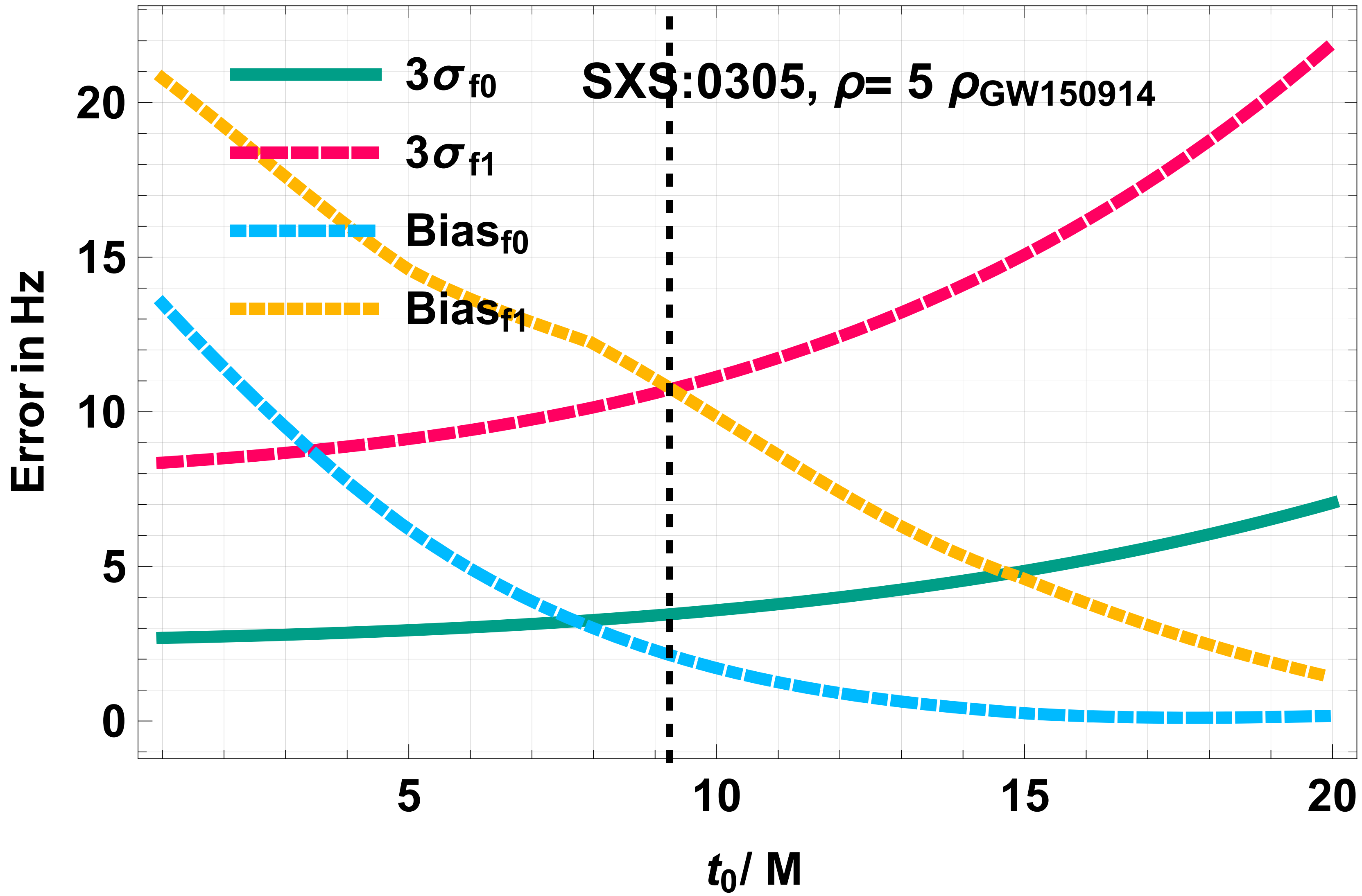

No, so long as we have the frequency of n=0 mode, we can model RD accurately with nonQNM. Then, can we interpret the damped sinusoids as QNMs of the BH?!?

# Picking the start time: 2-tone model

$$h = \sum_n C_n e^{\iota(t-t^0)\omega_n(1+\frac{\alpha_n}{100})} e^{-(t-t^0)/\tau_n}$$

Optimal start time is when Statistical Error = Systematic Bias

Berti et al. (Class. Quant. Grav. 26, 163001 (2009)) ⟶ $\rho\sigma_{f_i} = \kappa_i$

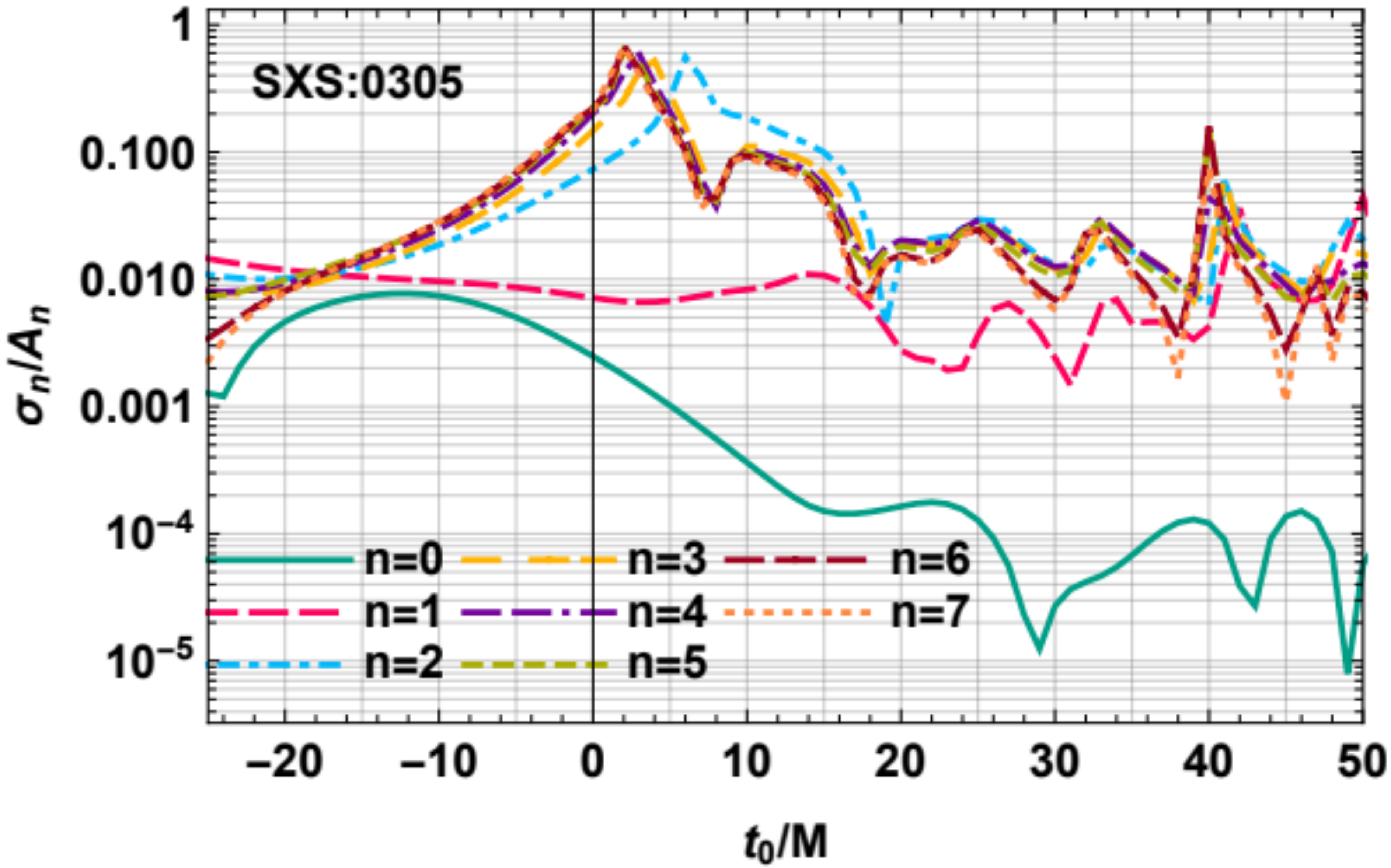


## However…

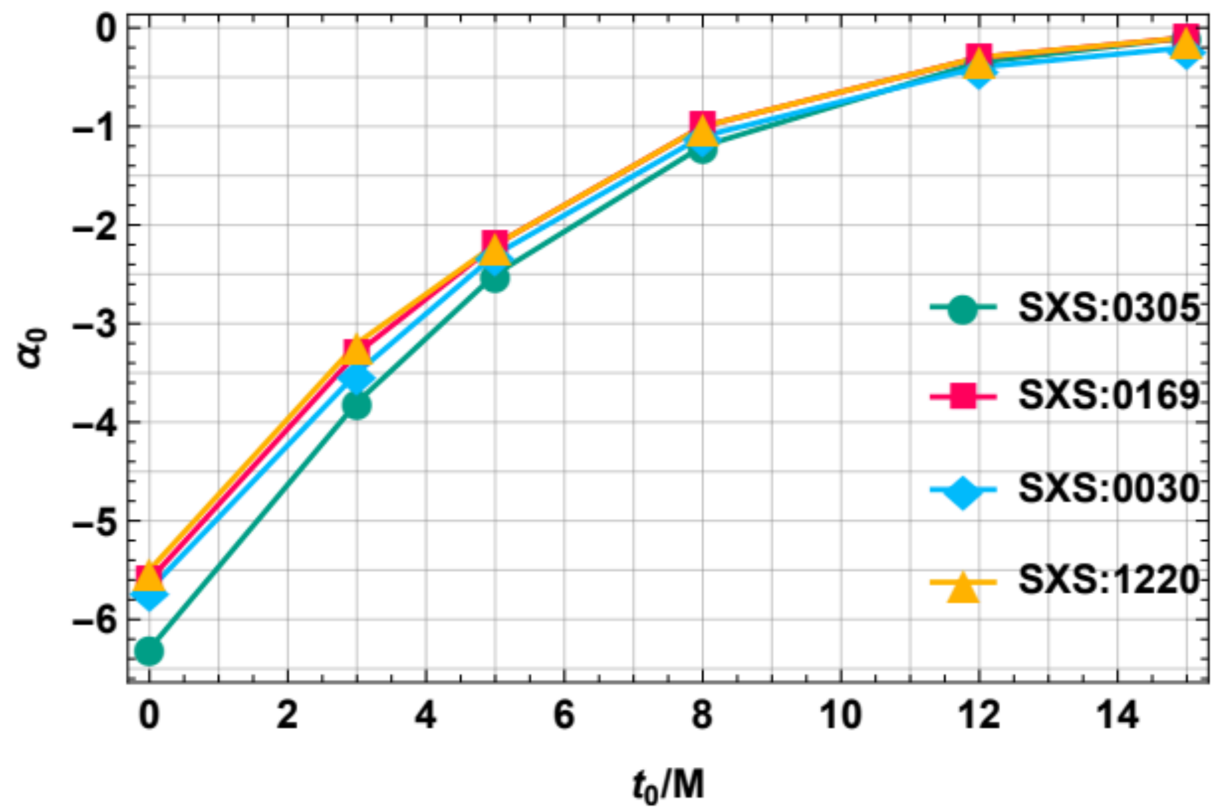
Independent of the interpretations of the damped sinusoids used in the modelling of RD, we can still test the no hair theorem using overtones from GW observations iff —

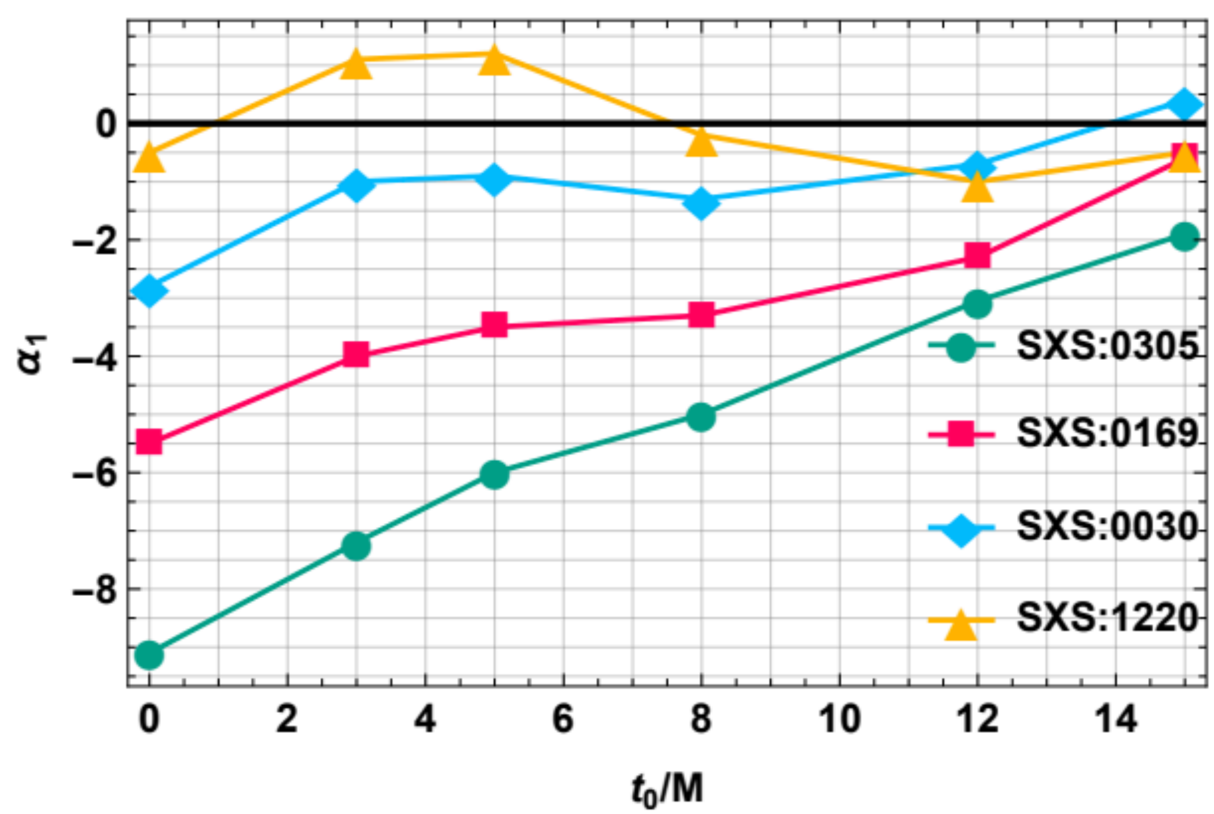
1. We recover the frequencies and damping time all QNMs from a Bayesian PE performed over ALL the RD model parameter.

2. The recover posterior for the frequencies and the damping time be such that the overtones frequencies and damping time be resolvable.



SXS:0305
n=0
n=1
n=2
n=3
n=4
n=5
n=6
n=7
$\sigma_n/\bar{A}_n$
$t_0$/M

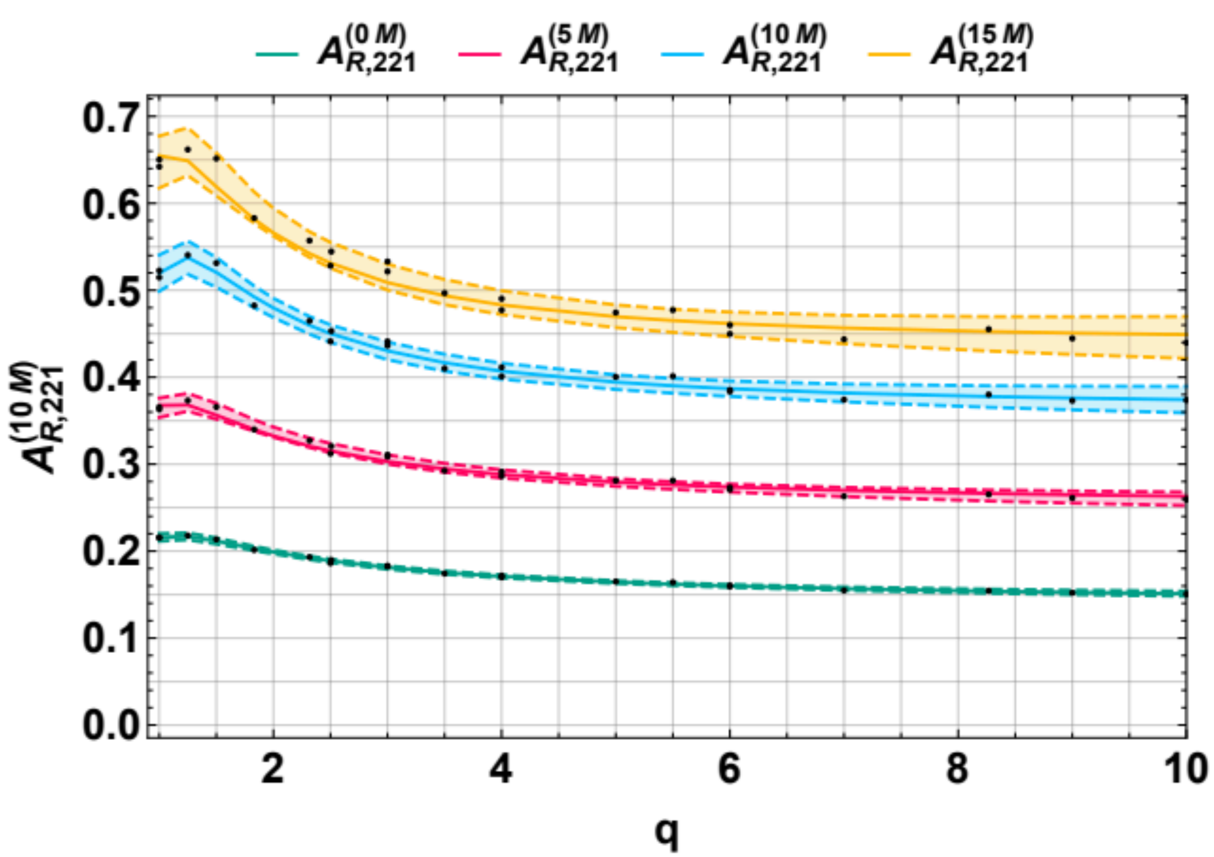

0
−1
−2
−3
−4
−5
−6
$\alpha_0$
0
2
4
6
8
10
12
14
$t_0$/M
SXS:0305
SXS:0169
SXS:0030
SXS:1220

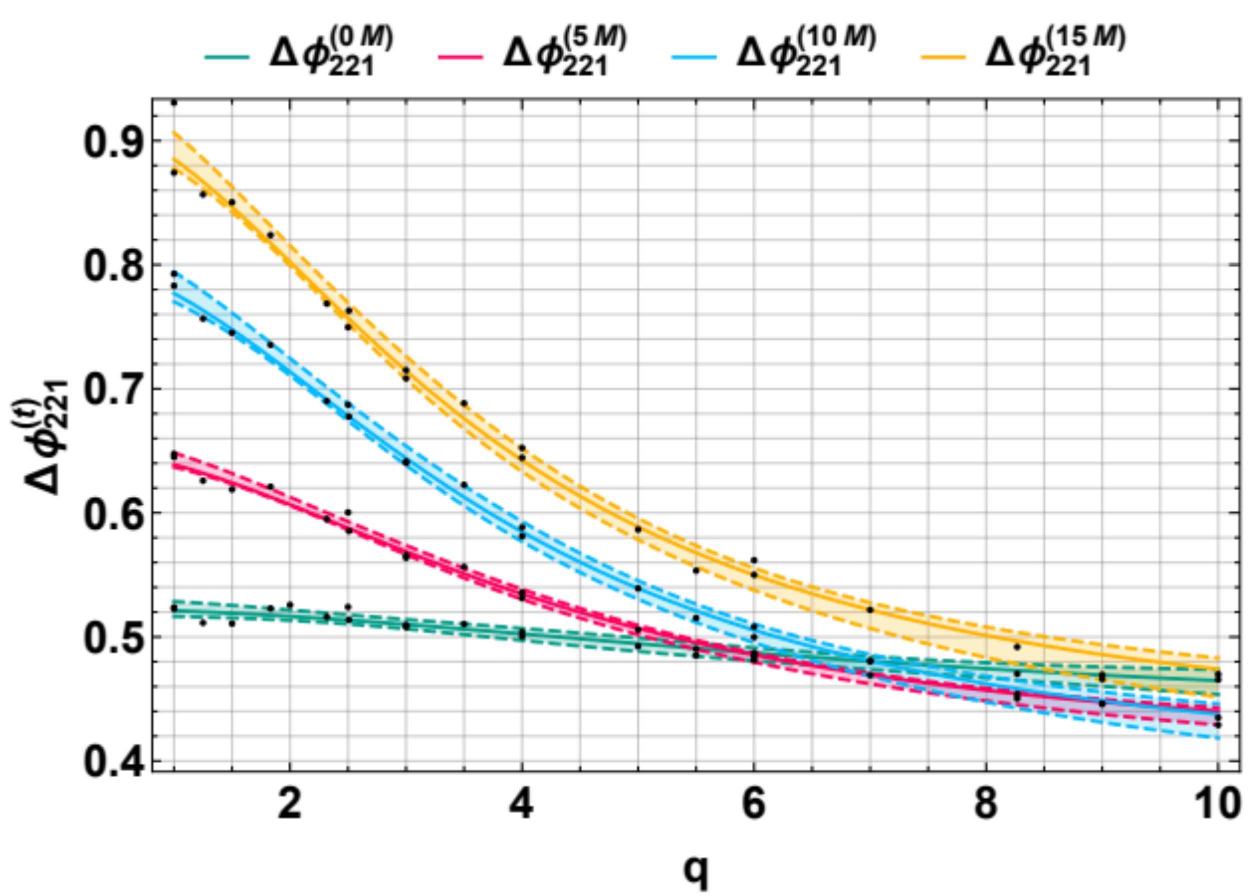

0
−2
−4
−6
−8
$\alpha_1$
0
2
4
6
8
10
12
14
$t_0$/M
SXS:0305
SXS:0169
SXS:0030
SXS:1220

# Symmetries

$$\begin{cases} t_0 \to t_0 + \Delta t \\ A_{lmn} \to A_{lmn} e^{-\frac{\Delta t}{\tau_{lnm}}} \\ \phi_{lnm} \to \phi_{lnm} \end{cases} .$$

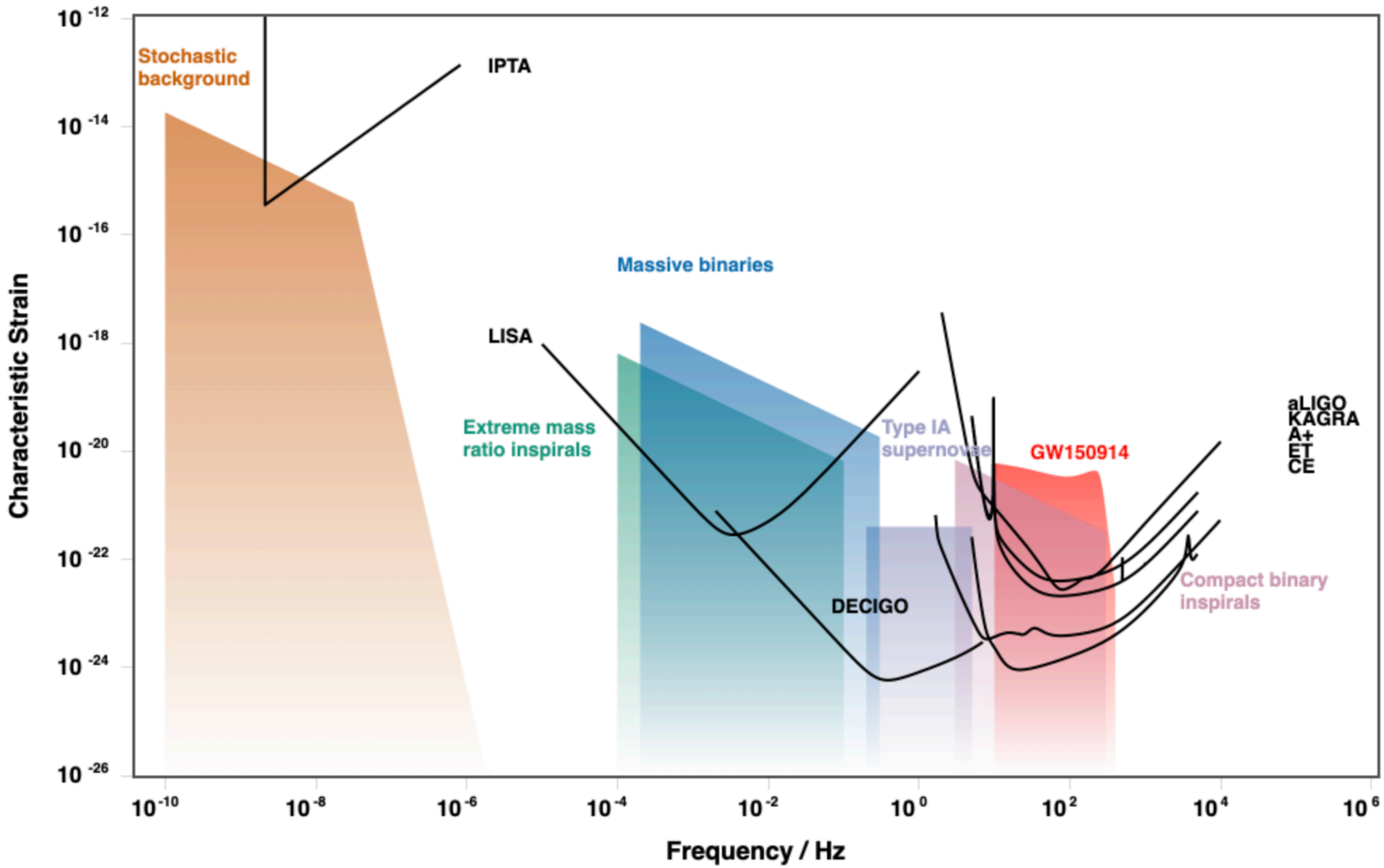
$$A_{R,lmn} = \frac{A_{lmn}}{A_{220}} \to A_{R,lmn} e^{-\Delta t (1/\tau_{lmn} - 1/\tau_{220})}$$




**Still some systematics!**

# On the Ringdown, overtones and BH spectroscopy




http://gwplotter.com/